Key

Math 285 ~~Sample Exam 2~~ Quiz 1 Version B

Instructions. The format of this exam is similar to the computer graded homework. You are encouraged to use scratch paper. There is limited partial credit. Only the answers you write inside the boxes will be graded.

1. Solve the given differential equation by separation of variables

$$\frac{dy}{dx} = e^{5x+3y}$$

to find the general solution.

$y(x) =$ $\boxed{-\frac{1}{3}\ln\left(C - \frac{3}{5}e^{5x}\right)}$

2. Find the solution of the linear differential equation

$$\begin{cases} \dfrac{dy}{dx} + y = e^{3x} \\ y(0) = 5 \end{cases}$$

The specific solution is

$y(x) =$ $\boxed{\frac{1}{4}e^{3x} + \frac{19}{4}e^{-x}}$

3. Determine whether the given differential equation is exact. If it is exact solve it; if it is not exact, enter NOT.

$$(2xy^2 - 5)dx + (2x^2y + 4)dy = 0$$

The answer

$\boxed{x^2y^2 - 5x + 4y = C}$

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4. Solve the given differential equation by finding an appropriate integrating factor.

$$4xy\,dx + (4y + 6x^2)dy = 0$$

The answer

$$\boxed{2x^2y^3 + y^4 = c.}$$

5. Solve the homogeneous differential equation by using an appropriate substitution.

$$(y^2 + yx)dx - x^2dy = 0.$$

The answer

$$\boxed{y = \frac{-x}{\ln|x| + c} \quad \text{or} \quad \frac{-x}{y} = \ln|x| + c}$$

6. Solve the givien initial-value problem. This is a Bernoulli equation.

$$\begin{cases} x^2\dfrac{dy}{dx} - 2xy = 5y^4 \\ y(1) = \dfrac{1}{3} \end{cases}$$

The answer

$$\boxed{\frac{1}{y^3} = \frac{-3}{x} + \frac{30}{x^6} \quad \text{or} \quad y = x^2(30 - 3x^5)^{-1/3}}$$

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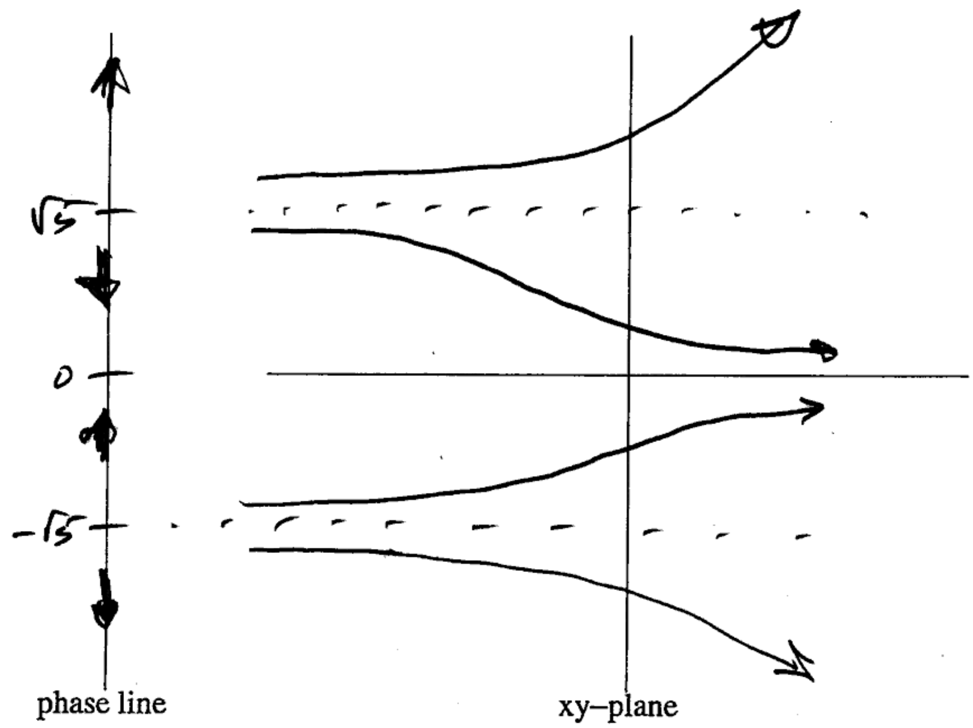
7. Draw a phase portrait and solution curves for the autonomous first-order ordinary differential equation $y' = y^3 - 5y$ below.



$$y^3 - 5y = y(y^2-5) = y(y-\sqrt{5})(y+\sqrt{5})$$

$$y = 0, -\sqrt{5}, \sqrt{5}$$

#1. $\frac{dy}{dx} = e^{5x}e^{3y}$; $\int e^{-3y}\,dy = \int e^{5x}\,dx$

$\frac{1}{-3}e^{-3y} = \frac{1}{5}e^{5x} + C$; $e^{-3y} = -\frac{3}{5}e^{5x} + c$

$-3y = \ln\left(-\frac{3}{5}e^{5x} + c\right)$; $y = -\frac{1}{3}\ln\left(c - \frac{3}{5}e^{5x}\right)$

#2 $y' + y = e^{3x}$; $(ye^{x})' = e^{4x}$; $ye^{x} = \frac{1}{4}e^{4x} + C$

$y = \frac{1}{4}e^{3x} + Ce^{-x}$; $5 = y(0) = \frac{1}{4} + C$; $C = 5 - \frac{1}{4} = \frac{19}{4}$

$y = \frac{1}{4}e^{3x} + \frac{19}{4}e^{-x}$.

#3 $(2xy^2 - 5)\,dx + (2x^2y + 4)\,dy = 0$; $M = 2xy^2 - 5$; $N = 2x^2y + 4$

$\frac{\partial M}{\partial y} = 4xy$; $\frac{\partial N}{\partial x} = 4xy$ ← same so exact.

$h(x,y) = \int (2xy^2 - 5)\,dx = x^2y^2 - 5x + g(y)$

$\frac{\partial h}{\partial y} = 2x^2y + g'(y) = 2x^2y + 4$; $g'(y) = 4$; $g(y) = 4y + c$.

$h(x,y) = x^2y^2 - 5x + 4y + c$; soln: $x^2y^2 - 5x + 4y = C$.

#4 $4xy\,dx + (4y + 6x^2)\,dy = 0$

$$\frac{\partial}{\partial y}(\mu 4xy) = \mu_y 4xy + \mu 4x$$

$$\frac{\partial}{\partial x}(\mu(4y+6x^2)) = \underbrace{\mu_x(4y+6x^2)}_{=0} + \mu\, 12x$$

Assume $\mu_x = 0$ so that $\mu = \mu(y)$ only. Then

$$\frac{d\mu}{dy} 4\cancel{x}y + \mu 4\cancel{x} = \mu\, 12\cancel{x}$$

$$\frac{d\mu}{dy} \cancel{4}y = \overset{2}{\cancel{8}}\mu\,; \qquad \int \frac{d\mu}{\mu} = \int \frac{2\,dy}{y}$$

$$\ln|\mu| = 2\ln|y| + C; \quad \ln|\mu| = \ln y^2 + C$$

$$|\mu| = y^2 e^C; \quad \mu = Cy^2$$

Therefore multiply the original equation by y^2

$$4xy^3\,dx + (4y^3 + 6x^2y^2)\,dy = 0$$

$$h(x,y) = \int 4xy^3\,dx = 2x^2y^3 + g(y)$$

$$\frac{\partial h}{\partial y} = \cancel{6x^2y^2} + g'(y) = 4y^3 + \cancel{6x^2y^2}$$

$$g'(y) = 4y^3; \quad g(y) = y^4 + C$$

$h(x,y) = 2x^2y^3 + y^4 + C$; Sol'n: $2x^2y^3 + y^4 = C$.

#5 $(y^2+yx)dx - x^2 dy = 0 \qquad y' = \frac{y^2+yx}{x^2}$

Let $y = ux$; $\quad y' = u'x + u$

$\frac{y^2+yx}{x^2} = u'x + u$; $\quad \left(\frac{y}{x}\right)^2 + \left(\frac{y}{x}\right) = u'x + u$

$u^2 + \cancel{u} = u'x + \cancel{u}$; $\qquad \frac{du}{dx} x = u^2$

$\int \frac{du}{u^2} = \int \frac{dx}{x}$; $\qquad -u^{-1} = \ln|x| + C$

$-\frac{x}{y} = \ln|x| + C$; $\quad y = \frac{-x}{\ln|x| + C}$

#6 $x^2 y' - 2xy = 5y^4$; $\quad y' - \frac{2}{x} y = \frac{5}{x^2} y^4$; $\quad n = 4$

$u = y^{1-n} = y^{-3}$; $\quad u' = -3y^{-4} y'$

$u' = -3y^{-4}\left(\frac{2}{x} y + \frac{5}{x^2} y^4\right) = -\frac{6}{x} y^{-3} - \frac{15}{x^2}$

$u' + \frac{6}{x} u = -\frac{15}{x^2}$; mult by $e^{\int \frac{6}{x} dx} = x^6$

$x^6 u' + 6x^5 u = -15x^4$; $\quad (x^6 u)' = -15x^4$

$x^6 u = -3x^5 + C$; $\quad x^6 y^{-3} = -3x^5 + C$

$\frac{1}{y^3} = \frac{-3}{x} + \frac{C}{x^6}$; Solve for C using $y(1) = \frac{1}{3}$

$27 = -3 + C$; $\quad C = 30.$

$\frac{1}{y^3} = \frac{-3}{x} + \frac{30}{x^6}$; $\quad y^3 = \frac{x^6}{30 - 3x^5}$; $\quad y = \frac{x^2}{\sqrt[3]{30 - 3x^5}}$

$y = x^2 (30 - 3x^5)^{-1/3}.$