Math 285 Sample Exam Version A

Instructions. This quiz comes in two parts. The first part, consisting of questions 1 and 2, will be graded based on the work you show. Points may be deducted for an unclear or incomplete presentation of your work even if the final answer is correct. The second part consists of questions 3 through 7 and is similar to the computer graded homework with limited partial credit. On the second part only your final answer will be graded.

1. Find the specific solution to the differential equation

$$\begin{cases} \dfrac{dy}{dx} + 6x^5 y = x^5 \\ y(1) = 3. \end{cases}$$

Integrating factor $\mu = e^{\int 6x^5 dx} = e^{x^6}$. Multiply

$$\underbrace{y' e^{x^6} + 6x^5 e^{x^6} y}_{(ye^{x^6})'} = x^5 e^{x^6}$$

$$(ye^{x^6})' = x^5 e^{x^6}$$

Therefore

$$ye^{x^6} = \int x^5 e^{x^6} dx = \frac{1}{6} e^{x^6} + C$$

General solution

$$y = \frac{1}{6} + C e^{-x^6}$$

Specific solution, solve for C so that $y(1) = 3.$

$$y(1) = \frac{1}{6} + Ce^{-1} = 3, \quad Ce^{-1} = \frac{17}{6}, \quad C = \frac{17e}{6}$$

Therefore

$$y(x) = \frac{1}{6} + \frac{17}{6} e^{1-x^6}.$$

2. Find the specific solution to the differential equation

$$\begin{cases} \dfrac{d^2y}{dx^2} + 36y = x \\ y(0) = 6, \quad y'(0) = -3. \end{cases}$$

Substitute $y = e^{rx}$ to find the characteristic equation

$r^2 + 36 = 0 \qquad r = \pm 6i$

Therefore, solutions to the homogeneous equation are

$y_1 = \cos 6x$ and $y_2 = \sin 6x$.

Guess a particular solution

$y_p = Ax + B$

$y_p' = A$

$y_p'' = 0$

Plugging in and solving for A and B yields

$36(Ax + B) = x$

Therefore $A = 1/36$ and $B = 0$.

The general solution is

$y(t) = C_1 \cos 6x + C_2 \sin 6x + \frac{1}{36} x$

Solve for C_1 and C_2 to find the specific solution

$y(0) = C_1 = 6 \quad$ so $\quad C_1 = 6$

$y'(0) = 6C_2 + \frac{1}{36} = -3 \quad$ so $\quad C_2 = \frac{1}{6}\left(\frac{-107}{36}\right) = \frac{-107}{216}$

Therefore

$y(t) = 6 \cos 6x - \frac{107}{216} \sin 6x + \frac{1}{36} x$.

3. Find the general solution to the differential equation

$$\frac{dy}{dx} = (-2x + y)^2 - 7$$

by making the substitution $u = -2x + y$.

$y(x) =$ $\boxed{\dfrac{c_1(2x-3)e^{x^6} - 2x - 3}{c_1 e^{x^6} - 1}}$

4. Consider the differential equation

$$x^2y'' - 7xy' + 16y = 0$$

with solution $y_1(x) = x^4$. Use reduction of order or any other valid mathematical technique to find a second linearly independent solution y_2.

$y_2(x) =$ $\boxed{x^4 \ln(x)}$

5. The number $N(t)$ of people in a community who are exposed to a particular advertisement is governed by the logistic equation. Initially, $N(0) = 400$, and it is observed that $N(1) = 800$. Solve for $N(t)$ if it is predicted that the limiting number of people in the community who will see the advertisement is 40000.

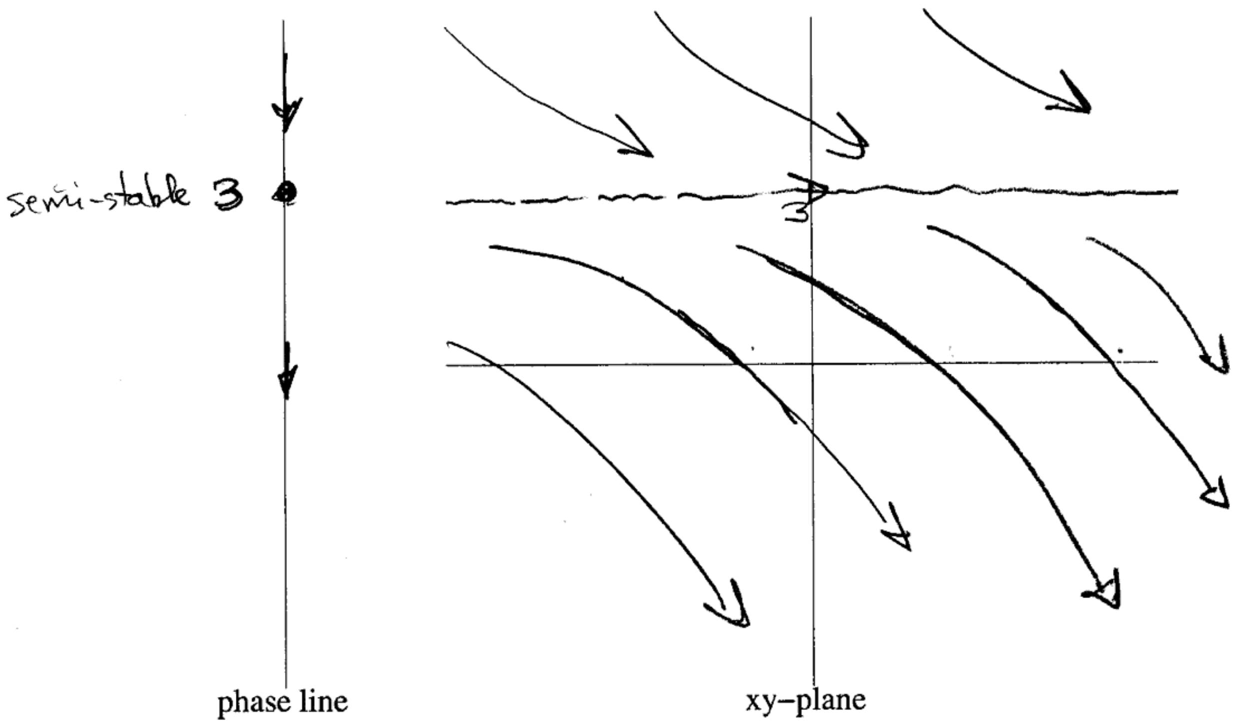
$N(t) =$ $\boxed{40000\left(\dfrac{1}{1+99e^{t\ln(49/99)}}\right)}$

6. Find the general solution to the differential equation

$$3y'' - 5y' + 2y = 0$$

$y(x) =$ $\boxed{c_1 e^{x} + c_2 e^{2x/3}}$

7. Draw a phase portrait and solution curves for the autonomous first-order ordinary differential equation $y' + y^2 + 9 = 6y$ below.



$$y' = -y^2 + 6y - 9 = -(y-3)^2$$