Course Summary for MATH 311

Note: Topics identified with DS are from *Introductory Real Analysis* by Frank Dangello and Michael Seyfried; topics identified with HE are from *Advanced Calculus: A Differential Forms Approach* by Harold Edwards.

1. Definitions
   i. DS: 6.1, 6.2, 6.3, 6.4, 6.5, 6.6, 6.8
   ii. HE: The definition of a compact oriented differentiable surface S in terms of an atlas $\{F_i\}_{i=1}^N$ (page 202abc).
   iii. HE: The definition of a continuous partition of unity $\{\phi_i\}_{i=1}^N$ with respect to an atlas $\{F_i\}_{i=1}^N$ (page 206).
   iv. HE: Definition for w to be a closed differential form and for w to be an exact differential form (pages 63 and 71).
2. Convergence and divergence of infinite series.
   i. DS: Examples 7.2, 7.3, 7.4, 7.7, 7.8, 7.10; homework §7.1#2,3,9 and §7.2#1,2.
3. Statement of Theorems
   i. DS: Statement of Theorem 8.10—The Weierstrass Approximation Theorem.
   ii. HE: Statement of Poincaré's Lemma (page 325).
   iii. HE: Statement of Stokes' Theorem (i.e., the Fundamental Theorem of Calculus) in the language of differential forms (page 217).
   iv. HE: The definition of $\int_S w$ where S is an compact oriented differentiable surface and w is a continuous 2-form on S as follows: Let S be a compact oriented differentiable surface and $\{F_i\}_{i=1}^N$ be an atlas for S. Let w be a continuous 2-form defined on S, and $\{\phi_i\}_{i=1}^N$ be a partition of unity with respect to the atlas $\{F_i\}_{i=1}^N$. Then by definition

$$\int_S w = \sum_{i=1}^N \int_R F_i^*(w_i)$$

   where $w_i = \phi_i w$ and $R = [-1,1] \times [-1,1]$.
4. DS: Example of a sequence of functions $f_n \in \mathcal{R}[a,b]$ that converge pointwise to a function $f \in \mathcal{R}[a,b]$ such that $\lim_{n\to\infty} \int_a^b f_n(x)dx \neq \int_a^b f(x)dx$.
5. HE: Differential Forms
   i. How to write with respect to the standard basis.
   ii. How to multiply them.
   iii. How to take the exterior derivative $d(w)$.
   iv. How to form the pullback $F^*(w)$.
6. HE: How to evaluate (integrate) constant 1-forms on directed line segments, constant 2-forms over oriented surface areas and constant 3-forms over oriented solids.
7. Proofs
   i. DS: Theorem 6.11 Fundamental Theorem of Calculus part 1.
   ii. DS: Theorem 6.12 Fundamental Theorem of Calculus part 2.
   iii. DS: Theorem 8.1 The uniform limit of continuous functions is continuous.
   iv. HE: Proof that the exterior derivative of a pullback is equal to the pullback of the exterior derivative $dF^*(w) = F^*(dw)$ in the case that w is a 1-form.