Math 330 Homework 3 Version A

1. Determine explicitly the following products of 3×3 elementary row matrices.

(i) $E_{12}E_{13}$

(ii) $E_2(5)E_{23}$

(iii) $E_{12}(-2)E_{13}(3)E_{23}$

(iv) $\big(E_{13}(100)\big)^{-1}$

(v) $\big(E_{12}(-2)E_{13}(3)E_{23}\big)^{-1}$

2. Let $A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$. Express A as a product of elementry row matrices.

3. A square matrix $D = [d_{ij}]$ is called diagonal if $d_{ij} = 0$ for $i \neq j$. Let $\text{diag}(a_1, \ldots, a_n)$ denote the $n \times n$ diagonal matrix with diagonal elements $d_{ii} = a_i$ for $i = 1, \ldots, n$. Let $D = \text{diag}(1, 2, 3, 6, 4)$. What is D^{-1}?

4. Find the number x for which the matrix

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & x \end{bmatrix}$$

is singular.

5. Let $A \in M_{n\times n}$.

(i) If $A^2 = 0$, prove that A is singular.

(ii) If $A^2 = A$ and $A \neq I$, prove that A is singular.

(iii) If $A^2 = I$, show that A is non-singular and find A^{-1}.

(iv) If $A^2 + 3A = I$, show that A is non-singular and find A^{-1}.

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6. Let

$$C = \begin{bmatrix} 1 & -2 & 3 \\ -1 & 2 & -2 \\ 2 & -2 & 4 \end{bmatrix}$$

Find C^{-1} by using the Gauss–Jordan algorithm to find the reduced row echelon form of

$$\begin{bmatrix} 1 & -2 & 3 & 1 & 0 & 0 \\ -1 & 2 & -2 & 0 & 1 & 0 \\ 2 & -2 & 4 & 0 & 0 & 1 \end{bmatrix}$$

Show all work and the row-operations needed.

7. Use Maple to find the reduced row echelon forms of the following matrices.

(i) $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \end{bmatrix}$

(ii) $\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 2 \\ 6 & 1 & 2 \\ 8 & 9 & 5 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 & 5 & -18 & 27 & 0 \\ -2 & \frac{1}{3} & -\frac{15}{3} & -\frac{7}{3} & 1 \\ 4 & 2 & 0 & 18 & -\frac{1}{2} \end{bmatrix}$

(v) $\begin{bmatrix} 1 & 12 & -3 & \frac{1}{2} \\ 2 & \sqrt{2} & 4 & -1 \\ -1 & 1 & 0 & 0 \end{bmatrix}$

8. Extra Credit: Work problems 7 and 10 from Mathews pages 50–51. The answers are written in the text already. Make sure your work explains in terms of detailed calculations how to get the answers.