Math 330 Homework 6 Version A

1. Find the least squares solution to $AX = B$ for the following matrices. Work these problems by hand.

   (i) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 15 \\ -7 \\ 0 \end{bmatrix}$.

   (ii) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 9 \\ 4 \\ 1 \\ -7 \\ 2 \\ -3 \end{bmatrix}$.

   (iii) $A = \begin{bmatrix} 1 & 4 & -4 \\ 2 & 3 & 2 \\ 5 & -1 & 0 \\ -1 & 5 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -11 \\ -2 \\ 7 \\ -9 \end{bmatrix}$.

2. Use the Maple subroutine `LeastSquares(A,B)` to check your answers to the least squares problems above. Include a printout of the output.

3. Let
$$u = 2 - \frac{1}{2}i, \quad v = 3 + i \quad \text{and} \quad w = 1 + 2i.$$

   Compute the following expressions and write your answers as a complex number of the form $a + bi$ where $a, b \in \mathbf{R}$.

   (i) $u - 3v$.

   (ii) uw.

   (iii) $(-i)^2 v$.

   (iv) v^{-1}.

   (v) $w\bar{w}$.

   (vi) e^i.

   (vii) $e^{\pi u}$.

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4. Find all solutions to

    (i) $z^2 + 4z + 5 = 0$.

    (ii) $z^2 + 5z + 13 = 0$.

    (iii) $z^2 + (1 + 2i)z + 3 - i = 0$.

    (iv) $z^3 - 5z^2 + 11z - 15 = 0$. Hint: $z = 3$ is one of them.

    (v) Extra Credit: $e^z = i$.

5. Find the reduced row-echelon form of the complex matrix

$$\begin{bmatrix} 2+i & -1+2i & 2 \\ 1+i & -1+i & 1 \\ 1+2i & -2+i & 1+i \end{bmatrix}.$$

6. Extra Credit: Let $S \subseteq \mathbf{R}^4$ be the subspace

$$S = \left\langle \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\rangle.$$

    (i) Is there a matrix A such that $\mathcal{N}(A) = S$?

    (ii) If so find it. If not explain why not.