Math 330 Homework 8 Version A

1. Let $A \in M_{n\times n}(\mathbf{R})$.

(i) Show that if λ is an eigenvalue of A then $\bar{\lambda}$ is also an eigenvalue of A.

(ii) If $A^t = A$, then show that the eigenvalues of A are real.

(iii) Let $B = A^t A$. Show that the eigenvalues of B are real and non-negative.

2. Use the Gram–Schmidt algorithm to find a set of orthonormal vectors that span the same space as the given vectors.

$$X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad X_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \qquad X_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}.$$

3. Let A and B be given by

$$A = \begin{bmatrix} 1 & 3 & 2 & 8 & 3 \\ 2 & 6 & 0 & -4 & -1 \\ -1 & -3 & -1 & -3 & 0 \\ 1 & 3 & 1 & 3 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 \\ -3 \\ -4 \\ 4 \end{bmatrix}.$$

(i) Find $\dim \mathcal{C}(A)$ and a basis for $\mathcal{C}(A)$.

(ii) Find $\dim \mathcal{N}(A)$ and a basis for $\mathcal{N}(A)$.

(iii) Find all solution to the equations $AX = B$.

4. Let A and B be given by

$$A = \begin{bmatrix} 1 & 3 & 0 \\ -1 & 0 & 1 \\ 1 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

(i) Show that $A^t A$ is diagonal.

(ii) Solve the least squares problem $AX = B$.

5. Let A by the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 7 & 8 \\ 0 & 0 & 9 & 1 & 2 \\ 0 & 3 & 4 & 5 & 6 \end{bmatrix}$$

(i) Find $\det A$.

(ii) Find $\det 2A$.

(iii) Find $\det(A - I)$.

(iv) find $\det A^{-1}$.

6. Let A be given by

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 3 & 1 \end{bmatrix}.$$

(i) Write A as a product of elementary row operations.

(ii) Find the inverse A^{-1} of A.

(iii) Verify that $AA^{-1} = I$.

7. If $A, B \in M_{n \times n}$ and $BA = I$ show that $AB = I$.

8. Choose five linear algebra terms that you think are important and carefully write out their definitions.

9. Extra Credit: Correct all errors in previous homework, quiz and exam problems.