Math 330 Midterm Version B

1. Determine whether the following pairs of vectors are perpendicular.

(i) Is $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ perpendicular to $\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$?

(ii) Is $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ penpendicular to $\begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$?

2. Let $A = \begin{bmatrix} 1 & 2 & 4 \\ -2 & 3 & 1 \\ -4 & 1 & 2 \end{bmatrix}$ and $v = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$.

(i) Compute $\|v\|$.

(ii) Compute Av.

3. Let $A \in \mathbf{R}^{m\times n}$ and $\mathcal{C}(A)$ be the columnspace of A. Then

(A) $\mathcal{C}(A) = \{\, Ax : x \in R^n \,\}$.

(B) $\mathcal{C}(A) = \{\, Ax : A \in R^m \,\}$.

(C) $\mathcal{C}(A) = \{\, x \in \mathbf{R}^n : Ax = 0 \,\}$.

(D) $\mathcal{C}(A) = \{\, x \in \mathbf{R}^m : Ax = 0 \,\}$.

(E) none of the above.

4. True or false with a counterexample if false and a reason if true.

(i) If $A \in \mathbf{R}^{n\times n}$ is invertible then A^2 is invertible.

(ii) If P is a permutation matrix corresponding to the row operation $r_i \leftrightarrow r_j$ where $i \neq j$ then $\mathcal{N}(P)$ is trivial.

(iii) If R is the reduced row echelon form of A then $\mathcal{C}(A) = \mathcal{C}(R)$.

5. Let

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{2} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find E^{-1}.

6. Let

$$A = \begin{bmatrix} 2 & 3 & 2 \\ 2 & 3 & 3 \\ 2 & 2 & 2 \end{bmatrix} \qquad \text{and} \qquad b = \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}.$$

Solve the system $Ax = b$. Find x.

7. Let $A \in \mathbf{R}^{4\times 7}$. Suppose A is a matrix that can be put into echelon form U using elimination without pivoting. How may row operations of the form $r_i \leftarrow r_i + \alpha r_j$ does the elimination algorithm take in general to put A into echelon form? Write these row operations in order.

8. Let $A \in \mathbf{R}^{m\times n}$ and $B \in \mathbf{R}^{l\times m}$. Prove that $(BA)^T = A^T B^T$.

9. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix} \qquad \text{and} \qquad b = \begin{bmatrix} -6 \\ -12 \\ -18 \end{bmatrix}$$

The nullspace of A is

$$\mathcal{N}(A) = \left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} c : c \in \mathbf{R} \right\}.$$

One solution to $Ax = b$ is

$$x = \begin{bmatrix} -1 \\ -10 \\ 5 \end{bmatrix}.$$

Find all solutions.

10. Let

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 2 & 2 & 0 & 3 \\ 0 & 6 & 5 & 4 \end{bmatrix}$$

Find the reduced row echelon form R of A.

11. Let

$$A = \begin{bmatrix} 1 & 2 & 5 & 6 & 2 \\ 2 & 4 & 4 & 2 & 8 \\ 6 & 12 & 1 & 1 & -2 \\ 4 & 8 & -3 & -1 & -10 \end{bmatrix}.$$

The reduced row echelon form of A is

$$R = \begin{bmatrix} 1 & 2 & 0 & 0 & -1/2 \\ 0 & 0 & 1 & 0 & 7/2 \\ 0 & 0 & 0 & 1 & -5/2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(i) Find $\dim(\mathcal{C}(A))$.

(ii) Find $\dim(\mathcal{N}(A))$.

(iii) Find a basis for $\mathcal{N}(A)$ and the nullspace matrix N corresponding to A.

12. Let $E_1, E_2 \in \mathbf{R}^{m\times m}$ be row operations of the form

$$E_1 = [r_i \leftarrow r_i + \alpha_1 r_j] \qquad \text{where} \qquad i \neq j$$

and

$$E_2 = [r_k \leftarrow r_k + \alpha_2 r_l] \qquad \text{where} \qquad k \neq l.$$

Is it always true that $E_1E_2 = E_2E_1$? If true explain why; if not provide a counter example where it is false.

13. Let

$$A = \begin{bmatrix} 1 & -1 & -2 \\ 2 & 1 & 2 \\ 4 & 1 & 8 \end{bmatrix}.$$

Find a lower triangular matrix L and an upper triangular U such that $LU = A$.