Key

Math 330 Quiz 1 Version A

**1.** Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & -3 \\ 1 & 3 & -1 \end{bmatrix}, \qquad x = \begin{bmatrix} 2 \\ 1 \\ -9 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}.$$

(i) Find $\frac{2}{3}b$ $= \frac{2}{3}\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2/3 \end{bmatrix}$

(ii) Find $x + b$ $= \begin{bmatrix} 2 \\ 1 \\ -9 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ -8 \end{bmatrix}$

(iii) Find $x \cdot b$ $= \begin{bmatrix} 2 \\ 1 \\ -9 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} = 6 + 0 - 9 = -3$

(iv) Find $Ax$ $= \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & -3 \\ 1 & 3 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -9 \end{bmatrix}$

$$= \begin{bmatrix} 2+2-27 \\ -2+0+27 \\ 2+3+9 \end{bmatrix} = \begin{bmatrix} -23 \\ 25 \\ 14 \end{bmatrix}$$

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**2.** Give a concrete example of a vector $v$ such that $v \in \mathbf{R}^4$.

$$v = \begin{bmatrix} 1 \\ 2 \\ 6 \\ -9.3 \end{bmatrix}$$

**3.** Give a concrete example of a matrix $A$ such that $A \in \mathbf{R}^{3\times 5}$.

$$A = \begin{bmatrix} 1 & 2 & 3 & 6 & -1 \\ 0 & 0 & 1 & 0 & 3 \\ 1 & 0 & 2 & -1 & 4 \end{bmatrix}$$

**4.** Let $u$ and $v$ be vectors in $\mathbf{R}^2$ such that $\|u\| = 1$ and $\|v\| = 1$. Use the angle addition and subtraction formulas from trigonometry to explain why $v \cdot u = \cos\theta$ where $\theta$ is the angle between the vectors $u$ and $v$.

Since $\|u\|=1$ and $\|v\|=1$ then $u$ and $v$ can be written in polar coordinates as

$$u = \begin{bmatrix} \cos a \\ \sin a \end{bmatrix} \quad v = \begin{bmatrix} \cos b \\ \sin b \end{bmatrix}$$

for some angles $a$ and $b$. Then the angle $\theta$ between $u$ and $v$ is $\theta = a-b$. It follows from the angle subtraction formula for cosine that

$$\cos\theta = \cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$= \begin{bmatrix} \cos a \\ \sin a \end{bmatrix} \cdot \begin{bmatrix} \cos b \\ \sin b \end{bmatrix} = u \cdot v.$$

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**5.** Apply the elimination algorithm to the matrix

$$\begin{bmatrix} 2 & 4 & 2 & -4 & 11 & 3 \\ 2 & 4 & 2 & -6 & 12 & 7 \\ -2 & -4 & -2 & 5 & -11 & -1 \end{bmatrix}$$

Indicate each row operation in the form $r_i \leftarrow r_i + \alpha r_j$ where $i \neq j$ and the write matrix after each operation.

$r_2 \leftarrow r_2 - r_1$

$$\begin{bmatrix} 2 & 4 & 2 & -4 & 11 & 3 \\ 0 & 0 & 0 & -2 & 1 & 4 \\ -2 & -4 & -2 & 5 & -11 & -1 \end{bmatrix}$$

$r_3 \leftarrow r_3 + r_1$

$$\begin{bmatrix} 2 & 4 & 2 & -4 & 11 & 3 \\ 0 & 0 & 0 & -2 & 1 & 4 \\ 0 & 0 & 0 & 1 & 0 & 2 \end{bmatrix}$$

$r_3 \leftarrow r_3 + \frac{1}{2} r_2$

$$\begin{bmatrix} 2 & 4 & 2 & -4 & 11 & 3 \\ 0 & 0 & 0 & -2 & 1 & 4 \\ 0 & 0 & 0 & 0 & 1/2 & 4 \end{bmatrix}$$

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**6.** Let $A \in \mathbb{R}^{4\times 12}$.

**(i)** Find $P$ such that the row operation $r_1 \leftrightarrow r_2$ on $A$ is given by $PA$.

$$P = \left[\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

**(ii)** Find $E$ such that the row operation $r_2 \leftarrow r_2 + \frac{1}{2}r_1$ on $AP$ is given by $E(PA)$.

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**(iii)** Compute the matrix $EP$. Multiply on right by $P$ is the column operation $C_1 \leftrightarrow C_2$

$$EP = \left[\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ 1 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

**(iv)** Find $D$ such that the row operation $r_3 \leftarrow 5r_3$ on $A$ is given by $DA$.

$$D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$