Math 330 Quiz 2 Version A

1. Let

$$A = \begin{bmatrix} 3 & 1 \\ 0 & 0 \\ 1 & 0 \\ -2 & 5 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 0 & -2 & -2 \\ 2 & 0 & -1 & 4 \end{bmatrix}$$

(i) Let $\mathcal{C}(A)$ be the column space of the matrix A. Then

(A) $\mathcal{C}(A) \subseteq \mathbf{R}^2$.

(B) $\mathcal{C}(A) \subseteq \mathbf{R}^3$.

(C) $\mathcal{C}(A) \subseteq \mathbf{R}^4$.

(D) none of the above.

(ii) Let $\mathcal{C}(A^T)$ be the column space of the matrix A^T. Then

(A) $\mathcal{C}(A^T) \subseteq \mathbf{R}^2$.

(B) $\mathcal{C}(A^T) \subseteq \mathbf{R}^3$.

(C) $\mathcal{C}(A^T) \subseteq \mathbf{R}^4$.

(D) none of the above.

(iii) Find A^T

(iv) Find BA

2. Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad \text{and} \qquad b = \begin{bmatrix} 7 \\ -4 \end{bmatrix}.$$

Find the vector x such that $Ax = b$.

3. A function $f : \mathbf{R}^n \to \mathbf{R}^m$ is said to be linear if

(A) $f(u + v) = f(u) + f(v)$ for every $u, v \in \mathbf{R}^n$.

(B) $f(\alpha u) = \alpha f(u)$ for every $u \in \mathbf{R}^n$ and $\alpha \in \mathbf{R}$.

(C) $f(x) = 0$ implies $x = 0$ for every $x \in \mathbf{R}^n$.

(D) both (A) and (B).

(E) both (A), (B) and (C).

4. A function f is said to be one-to-one if $f(u) = f(v)$ implies $u = v$ for every u and v in its domain. Let $A \in \mathbf{R}^{m\times n}$ and define $f\colon \mathbf{R}^n \to \mathbf{R}^m$ by $f(x) = Ax$. Show that f is one-to-one if and only if $f(x) = 0$ implies $x = 0$ for every $x \in \mathbf{R}^n$.

5. Let $A \in \mathbf{R}^{m\times n}$ and $\mathcal{N}(A)$ be the nullspace of A. Then

(A) $\mathcal{N}(A) = \{\, Ax : x \in R^n \,\}$.

(B) $\mathcal{N}(A) = \{\, Ax : A \in R^m \,\}$.

(C) $\mathcal{N}(A) = \{\, x \in \mathbf{R}^n : Ax = 0 \,\}$.

(D) $\mathcal{N}(A) = \{\, x \in \mathbf{R}^m : Ax = 0 \,\}$.

(E) none of the above.

6. Let $A = LU$ where

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ \frac{2}{3} & -\frac{4}{9} & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 5 & 6 \\ 0 & 0 & 0 & \frac{1}{7} \end{bmatrix}.$$

Find $\mathcal{N}(A)$.

7. Let

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix}.$$

(i) What eliminations matrices E_1, E_2 and E_3 transform A so $U = E_3E_2E_1A$ is in upper triangular or eschelon form?

(ii) Find L so that $A = LU$.