Key

Math 330 Quiz 2 Version A

1. Let

$$A = \begin{bmatrix} 3 & 1 \\ 0 & 0 \\ 1 & 0 \\ -2 & 5 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 0 & -2 & -2 \\ 2 & 0 & -1 & 4 \end{bmatrix}$$

(i) Let $\mathcal{C}(A)$ be the column space of the matrix A. Then

(A) $\mathcal{C}(A) \subseteq \mathbf{R}^2$.
(B) $\mathcal{C}(A) \subseteq \mathbf{R}^3$.
(C) $\mathcal{C}(A) \subseteq \mathbf{R}^4$.
(D) none of the above.

(ii) Let $\mathcal{C}(A^T)$ be the column space of the matrix A^T. Then

(A) $\mathcal{C}(A^T) \subseteq \mathbf{R}^2$.
(B) $\mathcal{C}(A^T) \subseteq \mathbf{R}^3$.
(C) $\mathcal{C}(A^T) \subseteq \mathbf{R}^4$.
(D) none of the above.

(iii) Find A^T

$$A^T = \begin{bmatrix} 3 & 0 & 1 & -2 \\ 1 & 0 & 0 & 5 \end{bmatrix}$$

(iv) Find BA

$$\begin{bmatrix} 3 & 1 \\ 0 & 0 \\ 1 & 0 \\ -2 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 0 & -2 & -2 \\ 2 & 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ -1 & -11 \\ -3 & 22 \end{bmatrix} = BA$$

2. Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 7 \\ -4 \end{bmatrix}.$$

Find the vector x such that $Ax = b$.

$$[A|b] = \begin{bmatrix} 1 & 2 & 7 \\ 3 & 4 & -4 \end{bmatrix}$$

$$r_2 \Leftarrow r_2 - 3r_1$$

$$\begin{bmatrix} 1 & 2 & 7 \\ 0 & -2 & -25 \end{bmatrix}$$

Thus

$$x_1 + 2x_2 = 7$$
$$-2x_2 = -25$$

Backsubstitution

$$x_2 = \frac{25}{2}$$

$$x_1 = 7 - 2x_2$$
$$= 7 - 25$$
$$= -18$$

Solution

$$x = \begin{bmatrix} -18 \\ 25/2 \end{bmatrix}.$$

3. A function $f : \mathbf{R}^m \to \mathbf{R}^n$ is said to be linear if

(A) $f(u+v) = f(u) + f(v)$ for every $u, v \in \mathbf{R}^m$.

(B) $f(\alpha u) = \alpha f(u)$ for every $u \in \mathbf{R}^m$ and $\alpha \in \mathbf{R}$.

(C) $f(x) = 0$ implies $x = 0$ for every $x \in \mathbf{R}^m$.

(D) both (A) and (B).

(E) both (A), (B) and (C).

4. A function f is said to be one-to-one if $f(u) = f(v)$ implies $u = v$ for every u and v in its domain. Let $A \in \mathbf{R}^{m \times n}$ and define $f: \mathbf{R}^n \to \mathbf{R}^m$ by $f(x) = Ax$. Show that f is one-to-one if and only if $f(x) = 0$ implies $x = 0$ for every $x \in \mathbf{R}^n$.

Suppose $f(u) = f(v)$ implies $u = v$. Let $f(x) = 0$. Since $f(x) = A0 = 0$ then $f(x) = f(0)$. Identifying $u = x$ and $v = 0$ we obtain that $x = 0$.

Conversely, suppose $f(x) = 0$ implies $x = 0$. Let $f(u) = f(v)$. Then $f(u-v) = A(u-v) = Au - Av = f(u) - f(v) = 0$. Now identifying $x = u - v$ we obtain $u - v = 0$ or that $u = v$.

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5. Let $A \in \mathbf{R}^{m\times n}$ and $\mathcal{N}(A)$ be the nullspace of A. Then

(A) $\mathcal{N}(A) = \{ Ax : x \in R^n \}$.

(B) $\mathcal{N}(A) = \{ Ax : A \in R^m \}$.

(C) $\mathcal{N}(A) = \{ x \in \mathbf{R}^n : Ax = 0 \}$.

(D) $\mathcal{N}(A) = \{ x \in \mathbf{R}^m : Ax = 0 \}$.

(E) none of the above.

6. Let $A = LU$ where

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ \frac{2}{3} & -\frac{4}{9} & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 5 & 6 \\ 0 & 0 & 0 & \frac{1}{7} \end{bmatrix}.$$

Find $\mathcal{N}(A)$.

Since $\mathcal{N}(A) = \mathcal{N}(U)$ we solve $Ux = 0$.

$$x_1 + 2x_2 + 3x_3 + 4x_4 = 0$$
$$5x_3 + 6x_4 = 0$$
$$\tfrac{1}{7} x_4 = 0$$

Back substitution

$x_4 = 0$

$x_3 = 0$

$x_1 = -2x_2$

$$x = \begin{bmatrix} -2x_2 \\ x_2 \\ 0 \\ 0 \end{bmatrix} = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

Thus

$$\mathcal{N}(A) = \operatorname{span}\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\} = \mathcal{C}\left(\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right).$$

7. Let

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix}.$$

(i) What eliminations matrices E_1, E_2 and E_3 transform A so $U = E_3E_2E_1A$ is in upper triangular or eschelon form?

$r_2 \leftarrow r_2 - 2r_1$

$r_3 \leftarrow r_3 - 3r_1$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 4 & 2 \end{bmatrix}$$

$r_3 \leftarrow r_3 - 2r_2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}, E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

(ii) Find L so that $A = LU$.

$$L = E_1^{-1}E_2^{-1}E_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}.$$