Linear Algebra Formula Sheet

Orthogonal Projection

$$P = A(A^T A)^{-1} A^T$$

Least Squares

$$x = (A^T A)^{-1} A^T b$$

Gram-Schmidt

$$\tilde{q}_1 = a_1 \qquad\qquad q_1 = \tilde{q}_1/\|\tilde{q}_1\|$$
$$\tilde{q}_2 = a_2 - q_1(q_1 \cdot a_2) \qquad\qquad q_2 = \tilde{q}_2/\|\tilde{q}_2\|$$
$$\tilde{q}_3 = a_3 - q_1(q_1 \cdot a_3) - q_2(q_2 \cdot a_3) \qquad\qquad q_3 = \tilde{q}_3/\|\tilde{q}_3\|$$
$$\vdots \qquad\qquad\qquad \vdots$$
$$\tilde{q}_n = a_n - q_1(q_1 \cdot a_n) - q_2(q_2 \cdot a_n) - \cdots - q_{n-1}(q_{n-1} \cdot a_n) \qquad q_n = \tilde{q}_n/\|\tilde{q}_n\|.$$

Least Squares using Gram-Schmidt

$$Rx = Q^T b \qquad \text{where} \qquad R = Q^T A$$

Combinatorial Determinant Formula

$$\det A = \sum_{\text{all } n \times n \text{ permuation matrices } P} (\det P)(\text{product of the diagonal of } PA)$$

Cofactor (Laplace's) Expansion

$$\det A = \sum_{j=1}^{n} a_{ij} C_{ij} \qquad \text{where} \qquad C_{ij} = (-1)^{i+j} \det M_{ij}$$

and M_{ij} is the submatrix of A resulting from deleting row i and column j.

Formula for the Inverse

$$A^{-1} = \frac{C^T}{\det(A)}$$

Cramer's Rule

$$x_1 = \frac{\det B_1}{\det A} \qquad x_2 = \frac{\det B_2}{\det A} \qquad . \quad . \quad . \qquad x_n = \frac{\det B_n}{\det A}$$