MATH 330: Problem Set 2

Due Friday, March 10, 2017

1. Determine if the system has a nontrivial solution (you do not need to completely solve the system).

$$\begin{aligned} x_1 - 3x_2 + 7x_3 &= 0 \\ -2x_1 + x_2 - 4x_3 &= 0 \\ x_1 + 2x_2 + 9x_3 &= 0 \end{aligned}$$

2. Write this system of equations as a matrix equation $A\vec{x} = \vec{b}$ and then find the complete solution of the system and write it in the form $\vec{x} = \vec{x}_p + \vec{x}_n$.

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 1 \\ -4x_1 - 9x_2 + 2x_3 &= -1 \\ -3x_2 - 6x_3 &= -3 \end{aligned}$$

3. Determine whether the vectors $\begin{bmatrix} 3 \\ -3 \\ 9 \end{bmatrix}$, $\begin{bmatrix} 8 \\ 0 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ -5 \\ 14 \end{bmatrix}$ are linearly independent.

4. Let $A = \begin{bmatrix} 2 & 0 & -5 \\ 3 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 8 & 4 \\ -7 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 & 4 \\ 1 & -1 & 6 \\ 0 & 2 & 2 \end{bmatrix}$, and $D = \begin{bmatrix} -4 & 3 & 1 \\ 9 & 8 & 2 \end{bmatrix}$. Determine whether the following operations are defined, and if so, compute them.

   (a) $A + B$

   (b) DB

   (c) BD

   (d) AC

   (e) $4A + 2D$

5. Use Gauss-Jordan elimination to find the inverse A^{-1} of the matrix $A = \begin{bmatrix} 1 & -4 & 1 \\ 7 & -5 & 3 \\ -2 & 2 & -1 \end{bmatrix}$.

6. Consider the vector space $\mathbb{R}^2$, whose vectors are column vectors with two entries $\begin{bmatrix} x \\ y \end{bmatrix}$, where $x, y \in \mathbb{R}$. Let H be the subset of $\mathbb{R}^2$ consisting of those vectors with entries that are required to be *integers*. Is H a subspace of $\mathbb{R}^2$? Why or why not? (*Hint:* Check the 3 subspace requirements.)

7. Find a condition on b_1, b_2, b_3 so that the following system is solvable:

$$\begin{bmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ -1 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

8. Let $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 0 & -1 \\ 4 & -4 & 7 \end{bmatrix}$. Find the rank of A and find its nullspace $N(A)$ by finding the special solutions to the equation $A\vec{x} = \vec{0}$.

9. Find a basis for the column space $C(A)$ for the matrix A in Problem 8.