Midterm Exam Review Sheet

MATH 330

1. Find the reduced echelon form of $A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 7 & 8 & 9 \end{bmatrix}$.

2. Solve the system

$$\begin{aligned} x_1 &+ 4x_2 &- 3x_3 &= 13 \\ 2x_1 &- x_2 &+ x_3 &= 0 \\ -3x_1 &+ 2x_2 &- x_3 &= -3. \end{aligned}$$

3. Find the complete solution to the system with augmented matrix and write it in the form $\vec{x} = \vec{x}_p + \vec{x}_n$.

$$\begin{bmatrix} 1 & 2 & -5 & -6 & 0 & -5 \\ 0 & 1 & -6 & -3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

4. Write a matrix equation that is equivalent to the system of equations

$$\begin{aligned} & & x_2 &+ 5x_3 &= 2 \\ 4x_1 &+ & 6x_2 &- x_3 &= 1 \\ -x_1 &+ & 3x_2 &- 8x_3 &= -8. \end{aligned}$$

5. Let $A = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$. Determine if $\vec{b}$ is a linear combination of the columns of A.

6. Let $\vec{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$, $\vec{a}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, and $\vec{a}_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}$. Determine if the vector $\vec{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$ is a linear combination of $\vec{a}_1$, $\vec{a}_2$, and $\vec{a}_3$ and if it is, find coefficients c_1, c_2, and c_3 and explicitly write a $\vec{b}$ as a linear combination $\vec{b} = c_1\vec{a}_1 + c_2\vec{a}_2 + c_3\vec{a}_3$.

7. Find the reduced echelon form of the matrix $\begin{bmatrix} 1 & 4 & -5 & 1 & 2 \\ 2 & 5 & -4 & -1 & 4 \\ -3 & -9 & 9 & 2 & 6 \end{bmatrix}$.

8. The augmented matrix is given for a system of equations. Determine if the system is consistent, and if it is, find the complete solution.

$$\begin{bmatrix} 1 & 2 & -3 & -8 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & 0 & 8 \end{bmatrix}$$

9. Let $\vec{u} = \begin{bmatrix} -9 \\ 7 \\ 3 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$. Find the following:

(a) $\vec{u} + \vec{v}$

(b) $\vec{v} - \vec{u}$

(c) $3\vec{v}$

(d) $-5\vec{u} + 2\vec{v}$

10. Compute the product or state that it is undefined.

(a) $\begin{bmatrix} 2 & -1 \\ 3 & 1 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 7 & -4 \\ -2 & 9 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 1 & 0 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ -8 \\ -1 \\ -3 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$

11. Let $A = \begin{bmatrix} 1 & -3 & 2 \\ -2 & 5 & -1 \\ 3 & -4 & 5 \end{bmatrix}$ and let $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. Determine if the equation $A\vec{x} = \vec{b}$ is consistent for all possible b_1, b_2, b_3. If the equation is not consistent for all possible choices, give a description of the set of all $\vec{b}$ for which the equation is consistent.

12. Let $A = \begin{bmatrix} 1 & -3 & 2 \\ -2 & 5 & -1 \\ 3 & -6 & -3 \end{bmatrix}$ and let $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. Determine if the equation $A\vec{x} = \vec{b}$ is consistent for all possible b_1, b_2, b_3. If the equation is not consistent for all possible choices, give a description of the set of all $\vec{b}$ for which the equation is consistent.

13. Solve the system given below and write your answer in vector form.

$$\begin{array}{rcrcrl} x_1 & + & 2x_2 & - & 3x_3 & = 0 \\ 4x_1 & + & 7x_2 & - & 9x_3 & = 0 \\ -x_1 & + & 2x_2 & + & 3x_3 & = 0. \end{array}$$

14. Describe all solutions of $A\vec{x} = \vec{b}$ and write your answer in vector form $\vec{x} = \vec{x}_p + \vec{x}_n$, where

$$A = \begin{bmatrix} 2 & -5 & 3 \\ -2 & 6 & -5 \\ -4 & 7 & 0 \end{bmatrix} \quad \text{and} \quad \vec{b} = \begin{bmatrix} -2 \\ 2 \\ 4 \end{bmatrix}.$$

15. Determine whether the columns of $A = \begin{bmatrix} -2 & 1 & 4 \\ 4 & 0 & -4 \\ -6 & 1 & 8 \end{bmatrix}$ are linearly independent.

16. Find the transpose of the matrix $\begin{bmatrix} 8 & 4 \\ -4 & 1 \\ -7 & 7 \end{bmatrix}$.

17. Determine if the two matrices are inverses of each other.

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 1 & -2 \\ 1 & 0 & -1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & -1 & 2 \\ -3 & -2 & 4 \\ -1 & 1 & 1 \end{bmatrix}$$

18. Use Gauss-Jordan elimination to find A^{-1}, if it exists.

(a) $A = \begin{bmatrix} 5 & -1 & 5 \\ 5 & 0 & 3 \\ 10 & -1 & 8 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 2 & 2 & 3 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 3 & 3 \\ 2 & 7 & 8 \end{bmatrix}$

19. Determine if the matrix $\begin{bmatrix} 6 & 5 & -6 \\ 7 & 2 & -7 \\ -4 & 0 & 4 \end{bmatrix}$ is invertible.

20. Find an LU decomposition for the matrix $A = \begin{bmatrix} 2 & 3 & 5 \\ 4 & 9 & 5 \\ 4 & -3 & 24 \end{bmatrix}$.

21. Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & -2 & 1 \\ -3 & 1 & 4 \end{bmatrix}$. Compute the determinant of A in the following ways:

(a) Via a cofactor expansion across the first row.

(b) Via a cofactor expansion down the second column.

(c) By row reducing A to an upper triangular matrix and multiplying the diagonal entries.

22. Let $A = \begin{bmatrix} 1 & 3 & 1 & 2 & 4 \\ -1 & -3 & 1 & 6 & -6 \\ 1 & 3 & 0 & -2 & 5 \end{bmatrix}$. Find the following:

(a) The rank of A.

(b) A basis for the column space $C(A)$.

(c) The dimension of the nullspace $N(A)$.

(d) A basis for the nullspace $N(A)$.

23. Let $A = \begin{bmatrix} -3 & 2 & 1 & 1 \\ 2 & -1 & 0 & 7 \\ -4 & 3 & 2 & 9 \\ 0 & 1 & 1 & -4 \end{bmatrix}$. Find the following:

(a) The rank of A.

(b) A basis for the column space $C(A)$.

(c) The dimension of the nullspace $N(A)$.

(d) A basis for the nullspace $N(A)$.

24. Consider the vector space of all 2×2 matrices $M_{2,2}$ and let H be the subset of $M_{2,2}$ that consists of all diagonal 2×2 matrices (i.e., matrices where the only possibly nonzero entries are on the main diagonal). Determine if H is a subspace of $M_{2,2}$. Show why or why not.