

Math 330: Sample Final Exam Version A

This exam is open book, open web-browser and open notes. Your desktop, web camera and audio are being remotely recorded by Proctorio. The difference between Quiz 1 and Quiz 2 is that Proctorio will be recording each of you individually rather than Zoom doing the same thing as part of a video conference call.

The following instructions have not changed; however, please read them anyway:

- Do not click on the I'm finished question in WebCampus until you have shown all your written work to web camera and clearly stated how many pages you will be scanning and turning in for later grading. There is no need to show any Julia work to the web camera because that has already been captured by the screen recorder.
- Starting now until after the semester ends on December 18 do not send email, text or any other type of message to anyone about questions appearing on this exam.
- If you have a paid membership to WolframAlpha, Chegg or other fee-based web service please log out of those services. As your screen will be recorded, also log out of bank accounts, personal email and online shopping sites.
- Make sure only one monitor is enabled on your computer. If you normally use two monitors, you will have to temporarily disconnect one of them during the exam as Proctorio can only record one screen at a time.
- Before starting the exam Proctorio will verify that your video, audio and desktop are properly being recording. You will then be asked to show your student identification and your desk and work environment to the web camera.
 - It is fine to have books, notes and blank paper on your desk.
 - It's better not to have any dogs, cats or other people in the room as the presence of multiple faces might confuse the computerized face detector.
 - If your favorite cat jumps on your desk during the exam do not panic; simply remove it and continue working.
- Work each problem using pencil and paper or using a computer and Julia as appropriate. It is not allowed to send or receive email during the exam or upload questions to any web forum or homework service.
 - It is fine to use Google and similar web search engines.
 - It is fine to use the free WolframAlpha. Do not log in to a paid account.
 - You may use any non-interactive web resources during the exam.
 - You may read Stack Overflow or the Julia website, but do not post any questions to these or any similar forums during the exam.
 - It is fine to open a browser tab to read the text book; however, I would recommend downloading a pdf copy ahead of time and using that if needed.
- If you find an error in the test or are confused about a question, please explain carefully in writing what is wrong and include that with your best attempt at an answer.

Math 330: Sample Final Exam Version A

- When you are finished
 - Make sure your pages are consecutively numbered.
 - State how many pages you will be turning in using your microphone.
 - Show your work one page at a time to the web camera.
 - Hold each page steady for a count of 10 so the web camera can focus on it.
 - There is no need to show any Julia work to the web camera because that has already been captured by the screen recorder.
- After you have shown all your written work to the web camera, return to WebCampus, answer the I'm finished question and press submit to stop the recording.
- After you have ended Proctorio you must still upload high-resolution scans of all work to Final Upload on WebCampus along with any Julia programs and computer output used to arrive at your final answers.
 - Upload all your written work as a single pdf file.
 - For Julia you may upload a JupyterLab notebook as a separate ipynb file.
 - Do not change anything before uploading copies of your work for grading.
 - Please type a note in the comment panel for the Final Upload if you notice a mistake in your work that you want to let me know about.

Except for 12(iii) which is extra credit, please answer all of the following questions:

1. Indicate in writing that you have understood the requirement to work independently by writing "I have worked independently on this exam" followed by your signature as the answer to this question.
2. Consider the matrices A and B given by

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 3 & 1 \\ -1 & 3 & -6 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 3 & -5 \\ 0 & 0 & 2 \end{bmatrix}$$

- (i) Explain in details why these matrices have the same determinant.
 - (ii) What is the common value of that determinant?
3. Consider the matrix

$$A = \begin{bmatrix} 2 & 3 \\ -2 & -4 \end{bmatrix}.$$

Write down the polynomial $p(\lambda) = \det(A - \lambda I)$.

4. Suppose $u, v \in \mathbf{R}^3$ are given by $u = (7, 2, -1)$ and $v = (-2, 3, 5)$.
 - (i) Find $v - 2v$.
 - (ii) Find vu^T .

Math 330: Sample Final Exam Version A

5. Let $x \in \mathbf{R}^{25}$ be such that $\|x\| = 8$. At most how many different entries of x could satisfy $|x_i| \geq 3$?

6. Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$$

- (i) Explain why the columns of this matrix must be linearly dependent.
(ii) Find a nonzero vector $c \in \mathbf{R}^3$ such that $Ac = 0$.

7. The matrix

$$A = \begin{bmatrix} 7 & 3 \\ -9 & -5 \end{bmatrix}$$

has eigenvalues $\lambda_1 = -2$ and $\lambda_2 = 4$.

- (i) Explain how to find eigenvectors corresponding to λ_1 and λ_2 by hand and then use a pencil and paper calculation to actually find them.
(ii) Check your answer with Julia.

8. Consider the over-determined system of linear equations $Ax = b$ where

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

For what value of x is $\|Ax - b\|$ minimized?

- (i) Explain how to find x using the QR factorization.
(ii) Explain how to find x using the pseudo-inverse A^\dagger .
(iii) Use Julia or pencil and paper to actually find x .

9. Suppose A , B and C are matrices that satisfy $AB = CC^T$.

- (i) Is it true or false that $A = C$ and $B = C^T$. If true explain why, if false provide a counter example.
(ii) Is it true or false that $AB = B^T A^T$. If true explain why, if false provide a counter example.

10. Consider the adjacency matrix

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

- (i) How many vertices are in the corresponding directed graph?
(ii) Draw a picture of the directed graph.
(iii) Starting at any given vertex is it possible to reach any other vertex in the graph?
(iv) Is there any matrix power A^k such that none of the entries of A^k are zero?

Math 330: Sample Final Exam Version A

- 11.** Suppose p is a polynomial of degree $n - 1$ or less given by $p(t) = c_1 + c_2t + \cdots + c_nt^{n-1}$. Let $q(t) = p(2t)$. Then $q(t) = d_1 + d_2t + \cdots + d_mt^{m-1}$ for some vector d . Find a matrix D for which $d = Dc$. Give the entries of D and be sure to specify its dimensions.
- 12.** Let $A = 2J - I$ where J is the reverser matrix

$$J = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 1 & 0 \\ \vdots & \vdots & . & \vdots & \vdots \\ 0 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

- (i) Show that $Ax = 0$ implies $x = 0$.
- (ii) Verify that $A^{-1} = \frac{1}{3}(2J + I)$.
- (iii) [Extra Credit] Find $\det(A)$ as a function of n .