Elimination step

ELEMENTARY ROW OPERATIONS

1. (Replacement) Replace one row (r_i) by the sum of itself and a multiple (α) of another row (r_j).[1]
2. (Interchange) Interchange two rows. r_i & r_j
3. (Scaling) Multiply all entries in a row by a nonzero constant.

① Elimination Step: $r_i \leftarrow \underbrace{r_i + \alpha r_j}$

Short hand $[r_i + \alpha r_j]$

assumes going back in r_i

② Row swap: $r_i \leftrightarrow r_j$

③ Scaling: $r_i \leftarrow \alpha r_i$ actually not a type of elimination step.

Example

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 6 \\ -1 & 0 & 8 \end{bmatrix} \begin{matrix} \leftarrow r_1 \\ \leftarrow r_2 \\ \leftarrow r_3 \end{matrix}$$

$r_2 \leftarrow r_2 - 2r_1$

$(\alpha = -2)$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 0 \\ -1 & 0 & 8 \end{bmatrix}$$

$r_3 \leftarrow r_3 + r_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 0 \\ 0 & 2 & 11 \end{bmatrix}$$

$r_3 \leftarrow r_3 + 2r_2$

diagonal stripe

$$U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

Note the order of the elimination steps preserves the zero's made earlier.

↑ Stop here if making matrix factorization
$A = LU$

Name is the Eschelon Form

How... Algorithm called Gaussian Elimination

Reduced Eschelon Form (keep going)...

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 0 \\ 0 & 0 & 11 \end{bmatrix} \qquad r_1 \leftarrow r_1 + 2r_2$$

↑ work to do here

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & -1 & 0 \\ 0 & 0 & 11 \end{bmatrix} \qquad r_1 \leftarrow r_1 - \frac{3}{11} r_3$$

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & -1 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

Now scaling...

$r_2 \leftarrow (-1) r_2$

$r_3 \leftarrow \left(\frac{1}{11}\right) r_3$

Stop here if the goal is to find A^{-1} ← inverse matrix of A.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

reduced Row eschelon form of the matrix A.

All the Elementary Row operations are reversible...

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$r_3 \leftarrow 11 r_3$

$r_2 \leftarrow -1 r_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$r_1 \leftarrow r_1 + \frac{3}{11} r_3$$

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & -1 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$