USING ROW REDUCTION TO SOLVE A LINEAR SYSTEM

1. Write the augmented matrix of the system.
2. Use the row reduction algorithm to obtain an equivalent augmented matrix in echelon form. Decide whether the system is consistent. If there is no solution, stop; otherwise, go to the next step.
3. Continue row reduction to obtain the reduced echelon form.
4. Write the system of equations corresponding to the matrix obtained in step 3.
5. Rewrite each nonzero equation from step 4 so that its one basic variable is expressed in terms of any free variables appearing in the equation.

don't have any free vbls in this example

$$1x + 3y - z = 4$$
$$-1x - 2y + 3z = -1$$
$$2x + 1y - 5z = -2$$

Solve simultaneously for x, y & z.

1
$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 4 \\ -1 & -2 & 3 & -1 \\ 2 & 1 & -5 & -2 \end{array}\right]$$

$r_2 \leftarrow r_2 + r_1$
$r_3 \leftarrow r_3 - 2r_1$

$$\begin{bmatrix} 1 & 3 & -1 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & -5 & -3 & -10 \end{bmatrix}$$

$r_3 \leftarrow r_3 + 5r_2$

2
$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 7 & 5 \end{array}\right]$$

Consistent

what does this row echelon matrix mean?

$$\begin{aligned} 1x + 3y - z &= 4 \\ 1y + 2z &= 3 \\ 7z &= 5 \end{aligned}$$

backwards

Could solve by (back) substitution

$z = \frac{5}{7}$

$y = 3 - 2z = 3 - \frac{10}{7} = \frac{11}{7}$

$x = 4 - 3y + z = 4 - \frac{33}{7} + \frac{5}{7} = ?$

continuing
reduced row-eschelon
form

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 4 \\ 0 & \boxed{1} & 2 & 3 \\ 0 & 0 & 7 & 5 \end{array}\right]$$

$r_1 \leftarrow r_1 - 3r_2$

$$\begin{bmatrix} 1 & 0 & -7 & -5 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & \boxed{7} & 5 \end{bmatrix}$$

$r_1 \leftarrow r_1 + r_3$

$r_2 \leftarrow r_2 - \frac{2}{7} r_3$

$3 - \frac{2}{7}5 = \frac{21-10}{7} = \frac{11}{7}$

~~rescaling~~

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 11/7 \\ 0 & 0 & 7 & 5 \end{bmatrix}$$

$\begin{bmatrix} r_1 \leftarrow 1 \cdot r_1 \\ r_2 \leftarrow 1 \cdot r_2 \end{bmatrix}$

$r_3 \leftarrow \frac{1}{7} r_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 11/7 \\ 0 & 0 & 1 & 5/7 \end{bmatrix}$$

$$\left\{\begin{aligned} 1x &= 0 \\ 1y &= 11/7 \\ 1z &= 5/7 \end{aligned}\right.$$

answer

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 11/7 \\ 5/7 \end{bmatrix}$$