

Free variables & inconsistent systems

many soln.

opposites ...

no solution

Row echelon form of an augmented matrix

Free vbs: x_1 (F), x_3 (F)
 free vbs: x_2 (P), x_4 (P), x_5 (P)

$$\left[\begin{array}{ccccc|c} 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 7 & 2 \end{array} \right]$$

The corresponding system

systematic solve for the pivots

$$\begin{cases} x_2 + 2x_3 + 3x_4 + 4x_5 = 5 & 59 \\ x_4 + 2x_5 = 3 & -35 \\ 7x_5 = 2 & 24 \end{cases}$$

Solve for $x_5 = \frac{2}{7}$
 $x_4 = 3 - 2x_5 = 3 - \frac{4}{7} = \frac{17}{7}$

$$\begin{aligned} x_2 &= 5 - 2x_3 - 3x_4 - 4x_5 \\ &= 5 - 2x_3 - \frac{51}{7} - \frac{8}{7} \\ &= \frac{35 - 51 - 8}{7} - 2x_3 \\ &= -\frac{24}{7} - 2x_3 \end{aligned}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} x_1 \\ -24/7 - 2x_3 \\ x_3 \\ 17/7 \\ 2/7 \end{bmatrix} = \begin{bmatrix} 0 \\ -24/7 \\ 0 \\ 17/7 \\ 2/7 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_3$$

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 7 & 2 \end{bmatrix}$$

$$r_1 \leftarrow r_1 - 3r_2$$

$$-4 + \frac{4}{7} \cdot \frac{-29}{7}$$

$$3 - \frac{4}{7} = \frac{21-4}{7} = \frac{17}{7}$$

$$\begin{bmatrix} 0 & 1 & 2 & 0 & -2 & -4 \\ 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 7 & 2 \end{bmatrix}$$

$$r_1 \leftarrow r_1 + \frac{2}{7}r_3$$

$$r_2 \leftarrow r_2 - \frac{2}{7}r_3$$

Scaling

$$\begin{bmatrix} 0 & 1 & 2 & 0 & 0 & -24/7 \\ 0 & 0 & 0 & 1 & 0 & 17/7 \\ 0 & 0 & 0 & 0 & 7 & 2 \end{bmatrix}$$

$$r_3 \leftarrow \frac{1}{7}r_3$$

$$\begin{bmatrix} 0 & 1 & 2 & 0 & 0 & -24/7 \\ 0 & 0 & 0 & 1 & 0 & 17/7 \\ 0 & 0 & 0 & 0 & 1 & 2/7 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ -24/7 \\ 0 \\ 17/7 \\ 2/7 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_3$$

$$\begin{bmatrix} 0 & 1 & 2 & 0 & 0 & -24/7 \\ 0 & 0 & 0 & 1 & 0 & 17/7 \\ 0 & 0 & 0 & 0 & 1 & 2/7 \end{bmatrix}$$

$$x_2 + 2x_3 = -24/7$$

$$x_4 = 17/7$$

$$x_5 = 2/7$$

$$x_2 = -\frac{24}{7} - 2x_3$$

Inconsistent Systems

$$\left[\begin{array}{cccc|c} 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 4 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{array} \right]$$

↗ If this is square
then same # of eqs
as vbls... and likely a
unique solution...

Note: how likely
it happens that a
system has free vbls
and or a pivot on
the wrong side depends
on how many rows and cols. there are...

↑
if there is a pivot on
the other side of the
yellow line then its
inconsistent.