



Free Variables



Inconsistent systems

↑
multiple
solns.

↑
no soln
at all.

Start with row echelon form of augmented matrix.

$$\begin{array}{cccccc|c}
 \text{F} & \text{F} & \text{P} & \text{P} & \text{F} & \text{P} & \\
 \hline
 0 & 0 & 2 & 1 & 2 & 5 & 8 \\
 0 & 0 & 0 & 1 & 2 & 3 & 4 \\
 0 & 0 & 0 & 0 & 0 & -1 & -5 \\
 \hline
 x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & = ?
 \end{array}$$

work
backwards

$$2x_3 + x_4 + 2x_5 + 5x_6 = 8$$

$$x_4 + 2x_5 + 3x_6 = 4$$

$$-x_6 = -5$$

Solve by back
substitution

$$x_6 = 5$$

$$x_4 = 4 - 2x_5 - 3x_6 = 4 - 2x_5 - 15 = -11 - 2x_5$$

$$x_3 = \frac{1}{2}(8 - x_4 - 2x_5 - 5x_6)$$

$$= \frac{1}{2}(8 - (-11 - 2x_5) - 2x_5 - 25)$$

$$= \frac{1}{2}(-6) = -3$$

Ans:

$$x_1 = x_1$$

$$x_2 = x_2$$

$$x_3 = -3$$

$$x_4 = -11 - 2x_5$$

$$x_5 = x_5$$

$$x_6 = 5$$

lots of
soln...

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -3 \\ -11 \\ 0 \\ 5 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix} x_5$$

Reduced Row Echelon Form \rightarrow

$$\begin{bmatrix} 0 & 0 & 2 & 1 & 2 & 5 & 8 \\ 0 & 0 & 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 & -1 & -5 \end{bmatrix}$$

$$r_1 \leftarrow r_1 - r_2$$

$$\begin{bmatrix} 0 & 0 & 2 & 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 & -1 & -5 \end{bmatrix}$$

$$r_1 \leftarrow r_1 + 2r_3$$

$$r_2 \leftarrow r_2 + 3r_3$$

rescale

$$\begin{bmatrix} 0 & 0 & 2 & 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 1 & 2 & 0 & -11 \\ 0 & 0 & 0 & 0 & 0 & -1 & -5 \end{bmatrix}$$

$$r_1 \leftarrow \frac{1}{2} r_1$$

$$r_3 \leftarrow -1 r_3$$

reduced row echelon form

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 & 2 & 0 & -11 \\ 0 & 0 & 0 & 0 & 0 & 1 & 5 \end{bmatrix}$$

] whats this eq?

$$x_4 + 2x_5 = -11$$

$$x_4 = -11 - 2x_5$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -3 \\ -11 \\ 0 \\ 5 \end{bmatrix} + x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ 0 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix}$$

Inconsistent Systems:

of Eqs

$$\left[\begin{array}{cccccc|c} 0 & 0 & 2 & 1 & 2 & 5 & 8 \\ 0 & 0 & 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & -5 \end{array} \right]$$

of vbls

If there is a pivot right of the yellow line then the system is inconsistent.

$0 = -5$ is a contradiction

→ NO SOLN!