System of linear equations:

$$3x - 5y + 2z = 7$$
$$2x + 3y - 1z = 3$$
$$-x - 7y + 3z = 1$$

Notation (so far):

linear function

$$f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 3x - 5y + 2z \\ 2x + 3y - 1z \\ -x - 7y + 3z \end{bmatrix}$$

rewrite the system using vectors (1.3).

$$\boxed{f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 7 \\ 3 \\ 1 \end{bmatrix}}$$

What is a lin. func?

① $f(X+Y) = f(X) + f(Y)$

② $f(\alpha X) = \alpha f(X)$

More notation:

$$f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 3x - 5y + 2z \\ 2x + 3y - 1z \\ -x - 7y + 3z \end{bmatrix} = \begin{bmatrix} 3 & -5 & 2 \\ 2 & 3 & -1 \\ -1 & -7 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

pattern...

holds by definition...

Let $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ then $\boxed{f(X) = AX}$

where $A = \begin{bmatrix} 3 & -5 & 2 \\ 2 & 3 & -1 \\ -1 & -7 & 3 \end{bmatrix}$

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Two way of viewing Matrix-vector mult

① row way

$$\begin{bmatrix} 3 & -5 & 2 \\ 2 & 3 & -1 \\ -1 & -7 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} (3,-5,2) \cdot (x,y,z) \\ (2,3,-1) \cdot (x,y,z) \\ (-1,-7,3) \cdot (x,y,z) \end{bmatrix}$$

② column way

$$\begin{bmatrix} 3 & -5 & 2 \\ 2 & 3 & -1 \\ -1 & -7 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix} x + \begin{bmatrix} -5 \\ 3 \\ -7 \end{bmatrix} y + \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} z$$

Solve $AX = b$

where $A = \begin{bmatrix} 3 & -5 & 2 \\ 2 & 3 & -1 \\ -1 & -7 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $b = \begin{bmatrix} 7 \\ 3 \\ 1 \end{bmatrix}$

How to solve? Factor $A$

$A = LU$ then $AX = b$ becomes

Solve $L(UX) = b$

$Y = UX$

Then I can solve

$$\begin{cases} LY = b \\ UX = Y \end{cases}$$

This is not more difficult to solve two equation, because the factoring of $A$ into $LU$ was done so $L$ and $U$ are matrices with a simpler structure...

What does $LU$ even mean?

We already know what

$$f(X) = AX$$

There fore I know what these function mean also

$$g(X) = UX$$

$$h(Y) = LY$$

Now if I take

$$(h \circ g)(X) = h(g(X)) = h(UX)$$

$$= L(UX) = \underbrace{(LU)}X$$

the matrix that represent the function $h \circ g$.

Matrix multiplication is exactly composition of the corresponding linear functions...

Is it clear the composition of two linear function is equal another lin. function that even has a matrix?

## Examples

simplest case $\begin{cases} g(x) = 7x \\ h(y) = 8y \end{cases}$

two lin. functions:

$$g(a+b) = 7(a+b) = 7a + 7b = g(a) + g(b)$$

Then what about the composition

$$(h \circ g)(x) = 8g(x) = 8(7x) = 56x$$

More convincing

Claim $(h \circ g)(a+b) = (h \circ g)(a) + (h \circ g)(b)$

claim $(h \circ g)(\alpha a) = \alpha (h \circ g)(a)$

$$\begin{aligned}(h \circ g)(a+b) &= h(g(a+b)) \\ &= h(g(a) + g(b)) \quad \text{since } g \text{ is lin.} \\ &= h(g(a)) + h(g(b)) \quad \text{since } h \text{ is lin} \\ &= (h \circ g)(a) + (h \circ g)(b) \quad \text{DONE!}\end{aligned}$$