Gaussian Elimination plays a big role in the first chapters...

Gaussian Elimination

Maybe already seen this

fluff

important why it linear alg works

Spiral method of teaching mathematics...keep switching back and forth between topics to give people a chance to catch up...

enough more in strands

We take a break discussing dimension and rank

Catch up with dimension and rank while learning determinant so everyone can continue together in chapter 4...

Learning more about dimension and rank...

Gaussian Elimination

Theory of Determinants

great stuff

Same thing happens here...

Take a break from the eigenvalue eigenvector problem to discuss Gram-Schmidt and least squares...

New algorithm

Stanford engineers

Finish line

generalization of eigenvector eigenvalue problem

This gives people time to catch up with the eigenvalue and eigenvectors before Chapter 7 which generalizes the eigenvalue eigenvector problem...to create the singular value decomposition...

While the spiral method at first looks like someone threw the pages for the book down the stairs and then put them back together in a random order...

The advertising claims that it allows those who fall behind time to catch up while other topics are being discussed while at the same time providing something new each time for people who keep pace with the course...

I'd prefer to cover one topic in depth before going on to the next topic... In either case, the thing to remember is that techniques get used again and again once learned, so it's never useful to forget a topic once the chapter is done...

$f(a+b) = f(a) + f(b)$

$f(\alpha a) = \alpha f(a)$

$f(x) = Ax$

THEOREM 5

If A is an $m \times n$ matrix, $\mathbf{u}$ and $\mathbf{v}$ are vectors in $\mathbb{R}^n$, and c is a scalar, then:

a. $A(\mathbf{u} + \mathbf{v}) = A\mathbf{u} + A\mathbf{v}$;

b. $A(c\mathbf{u}) = c(A\mathbf{u})$.

PROOF For simplicity, take $n = 3$, $A = [\,\mathbf{a}_1 \;\; \mathbf{a}_2 \;\; \mathbf{a}_3\,]$, and $\mathbf{u}, \mathbf{v}$ in $\mathbb{R}^3$. (The proof of the general case is similar.) For $i = 1, 2, 3$, let u_i and v_i be the ith entries in $\mathbf{u}$ and $\mathbf{v}$, respectively. To prove statement (a), compute $A(\mathbf{u} + \mathbf{v})$ as a linear combination of the columns of A using the entries in $\mathbf{u} + \mathbf{v}$ as weights.

$$\begin{bmatrix} u_1 + v_1 \end{bmatrix}$$

lin. comb of vectors with free vbls as the parameters...

$$X = \begin{bmatrix} 0 \\ -24/7 \\ 0 \\ 17/7 \\ 2/7 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_3$$

$$x = s\,u + t\,v$$

$$\begin{array}{ccccc} F & P & F & P & P \end{array}$$
$$\left[\begin{array}{ccccc|c} 0 & 1 & 2 & 0 & 0 & -24/7 \\ 0 & 0 & 0 & 1 & 0 & 17/7 \\ 0 & 0 & 0 & 0 & 1 & 2/7 \end{array}\right]$$

since in the homogeneous case the right-hand side starts zero then there is no const. vec. in the soln.

+0

$$\mathbf{x} = s\mathbf{u} + t\mathbf{v} \quad (s, t \text{ in } \mathbb{R})$$

to emphasize that the parameters vary over all real numbers. In Example 1, the equation

The set all vectors x of this form as s and t range over all real #'s is called a subspace...

TION

An indexed set of vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ in $\mathbb{R}^n$ is said to be **linearly independent** if the vector equation

$$x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + \cdots + x_p\mathbf{v}_p = \mathbf{0} \qquad Ax=0 \qquad (1)$$

has only the trivial solution. The set $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is said to be **linearly dependent** if there exist weights $c_1, \ldots, c_p$, not all zero, such that

$$c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_p\mathbf{v}_p = \mathbf{0} \qquad Ac=0 \qquad (2)$$

non zero solutio

put coef. in the matrix

Understand using matrix-vector mult...

$$A = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \cdots & \mathbf{v}_p \end{bmatrix}$$

then the eq. (1) is $Ax=0$

Use G. Elim. to find the (reduced) row eschelon form of A.

$$A = \begin{array}{cccccc} P & P & F & P & F & P \\ \end{array}$$
$$A = \begin{bmatrix} \square & * & * & * & * & * \\ 0 & \square & * & * & * & * \\ 0 & 0 & 0 & \square & * & * \\ 0 & 0 & 0 & 0 & 0 & \square \end{bmatrix}$$

Ask question: are there any free varibles? Two free vbls...

If only one soln. no free vbls...

If free vbls then many solns.

Lin. Dependent...

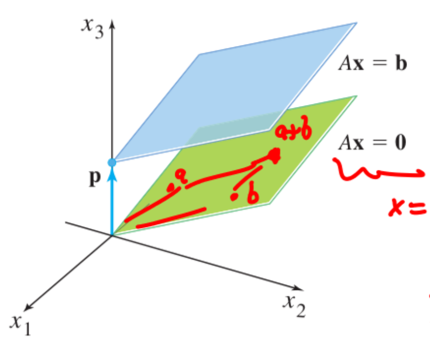

FIGURE 6 Parallel solution sets of $A\mathbf{x} = \mathbf{b}$ and $A\mathbf{x} = \mathbf{0}$.

$x = su + tv$

for s,t all real #'s

ie. $s,t \in \mathbb{R}$.

Subspaces are sets of points that form a plane or hyperplane which passes through the origin.