1.5 SOLUTION SETS OF LINEAR SYSTEMS

$2x_1 + 3x_2 = 0$
$-1x_1 + 2x_2 = 0$

Solve

$$\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Solution sets of linear systems a
will appear later in several differ
explicit and geometric descriptio

Homogeneous Linear

A system of linear equations is s
$\boxed{A\mathbf{x} = \mathbf{0}}$, where A is an $m \times n$ r
$A\mathbf{x} = \mathbf{0}$ *always* has at least one

Simplification:
zero on right hand side

$$\begin{array}{cccc} P & P & F & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{4}{3} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \begin{aligned} x_1 \quad - \tfrac{4}{3}x_3 &= 0 \\ x_2 \quad &= 0 \\ \boxed{0 \quad = 0} \end{aligned}$$

ok, consistent

$A\mathbf{x}=0$ is always consistent!

variables x_1 and x_2 and obtain $x_1 = \frac{4}{3}x_3$, $x_2 = 0$, with x_3 free. As a

THEOREM 6

Suppose the equation $A\mathbf{x} = \mathbf{b}$ is consistent for some given $\mathbf{b}$, and let $\mathbf{p}$ be a solution. Then the solution set of $A\mathbf{x} = \mathbf{b}$ is the set of all vectors of the form $\mathbf{w} = \mathbf{p} + \mathbf{v}_h$, where $\mathbf{v}_h$ is any solution of the homogeneous equation $A\mathbf{x} = \mathbf{0}$.

$f(x) = Ax$

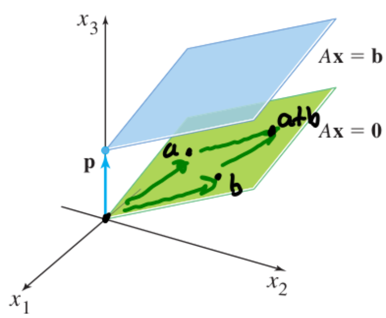
Theorem 6 says that if $A\mathbf{x} = \mathbf{b}$ has a solution, then the soluti
translating the solution set of $A\mathbf{x} = \mathbf{0}$, using any particular solut
the translation. Figure 6 illustrates the case in which there are two
when $n > 3$, our mental image of the solution set of a consistent s
$\mathbf{b} \neq \mathbf{0}$) is either a single nonzero point or a line or plane not passin

$Aa = 0 \qquad f(a) = 0$

$Ab = 0 \qquad f(b) = 0$

$f(a) + f(b) = f(a+b) = A(a+b) = 0$

$y = mx + b$



- the sum of vectors in the green plane is again a vector in the green plane.

Section 1.5 Homogeneous eq

(this col is zero.)

reduced row echelon form

$$
\begin{array}{ccccccc}
x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & \\
F & F & P & P & F & P & \\
\end{array}
$$

$$
\left[\begin{array}{cccccc|c}
0 & 0 & 1 & 0 & 0 & 0 & -3 \\
0 & 0 & 0 & 1 & 2 & 0 & -11 \\
0 & 0 & 0 & 0 & 0 & 1 & 5
\end{array}\right]
$$

first col → first element

4th element

3rd, 4th, 6th

$$
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ -3 \\ -11 \\ 0 \\ 5 \end{bmatrix}}_{p} + \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix} x_5}_{\text{soln to homogeneous equation. } Ax=0}
$$

· parametric
· vector
· form

Decompose soln. from before into two parts

p + soln to homogeneous equation. $Ax=0$

> An indexed set of vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ in $\mathbb{R}^n$ is said to be **linearly independent** if the vector equation
>
> $$x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + \cdots + x_p\mathbf{v}_p = \mathbf{0} \qquad Ax=0$$
>
> has only the trivial solution. The set $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is said to be **linearly dependent** if there exist weights $\boxed{c_1, \ldots, c_p, \text{ not all zero}}$ such that
>
> $$c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_p\mathbf{v}_p = \mathbf{0} \qquad Ac=0 \qquad (2)$$

[No free variables]

zero soln

[Means there are free vbls when solving $Ax=0$]

I think of c as being a soln. to the eq. $Ax=0$.

rewrite definition in the language of matrices

$$
\begin{array}{c} \xleftarrow{\quad p \text{ cols} \quad} \\ A = \left[\begin{array}{c|c|c|c} v_1 & v_2 & \ldots & v_p \end{array}\right] \end{array} \; \updownarrow q\text{-rows}
$$

how many rows? whatever the dimension (call it q) of the vectors $v_1, v_2, \ldots, v_p$.

how many cols? p

$$
x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_p \end{bmatrix} \qquad c = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_p \end{bmatrix}
$$

THEOREM 8 If a set contains more vectors than there are entries q in each vector, then the set is linearly dependent. That is, any set $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ in $\mathbb{R}^n$ is linearly dependent if $p > n$.

$p = 5, \quad q = 3$

$$\begin{array}{c c} & \longleftarrow 5 \longrightarrow \\ \updownarrow 3 & \begin{bmatrix} * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{bmatrix} \end{array}$$

look in each row of the eschelon form first non-zero entry is the pivot

(reduced)

$$\begin{bmatrix} \blacksquare & * & 0 & * & 0 \\ 0 & 0 & \blacksquare & * & 0 \\ 0 & 0 & 0 & 0 & \blacksquare \end{bmatrix}$$

Since there are only three rows, at most there are three pivots

Therefore there is always free vbls in this case... $p > q$.