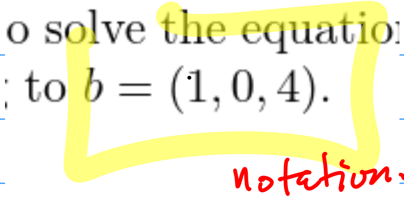
Notatation and a few other things from earlier in the book...



o solve the equatio

; to $b = (1, 0, 4)$.

writing a vector horizonally...

notation...

$$(1,0,4) = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$$

Note:

$$\begin{bmatrix} 1 & 0 & 4 \end{bmatrix} \neq \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$$

Transpose :

$$\begin{bmatrix} 1 & 0 & 4 \end{bmatrix}^T = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$$

means switch the rows with the columns...

(More in 2.1)

Example

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 6 & -1 & 4 & 0 \end{bmatrix}^T = \begin{bmatrix} 1 & 6 \\ 2 & -1 \\ 3 & 4 \\ 4 & 0 \end{bmatrix}$$

What is a transpose used for? what does it mean?

What did the matrix mean in the first place?

Given a linear function

$$f: \underset{\text{domain}}{\mathbb{R}^n} \to \underset{\text{range}}{\mathbb{R}^m}$$

$$f(x) = Ax \quad \text{where} \quad A \in \mathbb{R}^{m \times n}$$

rows cols

Consider a new function based on f:

$$g(z) = f(x) \cdot z$$

for this dot prod to make sense, length of z need to be the same as f(x)

because properties of dot products...

① is g(z) linear...

② what's its domain...

③ what's its range...

Thus $z \in \mathbb{R}^m$

The range of g is just a scalar, because that what dot products do.

$$g : \mathbb{R}^m \to \mathbb{R} \ldots$$

what is the matrix for g?

If $g(z) = Bz$

then $B = \left[g(e_1) \,|\, g(e_2) \,|\, \cdots \,|\, g(e_m) \right]$

$$g(e_1) = g\left(\begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \right) = f(x) \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$ = first component of the vector $f(x)$

$$g(z) = \boxed{f(x) \cdot z}$$

$g(e_2)$ = second component of $f(x)$

$\vdots$

$B = \left[\text{The components of } f(x) \text{ horizonally} \right] = f(x)^T$

Therefore $\boxed{g(z) = f(x)^T z}$ ← simple matrix vector mult.

The easy part of the story...

That's a little bit about transposes... but there's more...

Question: Can you find a function h so

$$f(x)\cdot z = x\cdot h(z)$$ for all x and z

Yes... how to find $h(z)$?

$f(x) = Ax$ it turns out $h(z) = A^T z$

(just a miracle)

recall

$f: \mathbb{R}^n \to \mathbb{R}^m$

$A \in \mathbb{R}^{m\times n}$

could figure it out by plugging in $e_1, e_2, e_3\ldots$ and so forth for x and z

Does $h(z) = A^T z$ make sense?

$$f(x)\cdot z = \overset{\mathbb{R}^n}{x}\cdot h(z)$$ for all x and z

okay...

does this dot prod make sense?

rows cols

$A^T \in \mathbb{R}^{n\times m}$

$h: \mathbb{R}^m \to \mathbb{R}^n$

$z \in \mathbb{R}^m$

another name for this is the adjoint

Thus $\boxed{Ax\cdot z = x\cdot A^T z}$