DEFINITION

A **subspace** of $\mathbb{R}^n$ is any set H in $\mathbb{R}^n$ that has three properties:

a. The zero vector is in H.

b. For each $\mathbf{u}$ and $\mathbf{v}$ in H, the sum $\mathbf{u}+\mathbf{v}$ is in H.

c. For each $\mathbf{u}$ in H and each scalar c, the vector $c\mathbf{u}$ is in H.

From (c) you could take the scalar $c=-1$ the $cu=-u$

Therefore $\boxed{-u \text{ is in } H.}$ could write this as $\boxed{-u \in H}$

From (b) you could then take $v=-u$ since that's in H

Therefore $u+v = u+(-u) = 0$ is in H.

There was an assumption that H contains any vector at all...

The main purpose of (a) is to guarantee H isn't the empty set.

outputs inputs

$A \in \mathbb{R}^{m \times n}$

ION

The **column space** of a matrix A is the set Col A of all linear combinations of the columns of A.

The same as the range of the function $f(x) = Ax$...

$$\text{range} f = \{ f(x) : x \in \mathbb{R}^n \}$$

$$\text{Col } A = \{ Ax : x \in \mathbb{R}^n \}$$

IITION

The **null space** of a matrix A is the set Nul A of all solutions of the homogeneous equation $A\mathbf{x} = \mathbf{0}$.

domain "such that"

$$\text{Nul } A = \{ x \in \mathbb{R}^n : Ax = 0 \}.$$

Elements in the set "in" Condition that the elements have to satisfy..

FINITION

A **basis** for a subspace H of $\mathbb{R}^n$ is a linearly independent set in H that spans H.

EXAMPLE Make a basis for Col A ...

$$\text{Col } A = \{ Ax : x \in \mathbb{R}^n \}$$

$$\boxed{A = \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix}} \qquad Ax = \underbrace{v_1x_1 + v_2x_2 + \cdots + v_nx_n}_{\text{definition of the span of the vectors } v_1, v_2, \ldots, v_n.}$$

$$\text{span}\{v_1, v_2, \ldots, v_n\} = \{ v_1x_1 + v_2x_2 + \cdots + v_nx_n : x \in \mathbb{R}^n \}$$

$$= \{ v_1x_1 + v_2x_2 + \cdots + v_nx_n : x_1, x_2, \ldots, x_n \in \mathbb{R} \}.$$

This is the same as Col A...

EM 13 The pivot columns of a matrix A form a basis for the column space of A.

Find the row-eschelon form...

$$A = \begin{bmatrix} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 \end{bmatrix} \xrightarrow{\text{Gauss elimination}} \begin{array}{ccccccccc} F & P & F & P & F & F & P & F & F \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 \end{array}$$

$$\begin{bmatrix} 0 & * & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & * & - & - & - & - & - \\ 0 & 0 & 0 & 0 & 0 & 0 & * & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Basis is $\underbrace{\{v_2, v_4, v_7\}}$

span

Therefore

$$\text{Col } A = \{ Ax : x \in \mathbb{R}^9 \} = \{ Bx : x \in \mathbb{R}^3 \} \text{ where } B = \begin{bmatrix} v_2 & v_4 & v_7 \end{bmatrix}$$

We'll discuss how to create a basis for the null space Nul(A) of the matrix A and then finish chapter 2. Please start reading chapter 3 about determinants...