DEFINITION The **dimension** of a nonzero subspace H, denoted by $\dim H$, is the number of vectors in any basis for H. The dimension of the zero subspace $\{\mathbf{0}\}$ is defined to be zero.[2]

DEFINITION The **rank** of a matrix A, denoted by rank A, is the dimension of the column space of A.

Since the basis of Col A was made from the pivot columns of A then the dimension of Col A :

$$\text{rank } A = \dim(\text{Col } A) = \# \text{ of pivots}$$

THEOREM 14 **The Rank Theorem**

If a matrix A has n columns, then $\text{rank } A + \dim \text{Nul } A = n$.

If $A \in \mathbb{R}^{m \times n}$ # of vbls in the eq..

$\underbrace{\text{\# of pivots} + \text{\# of free vbls}}_{} = n$

15 **The Basis Theorem**

use this hypothesis...

Let H be a p-dimensional subspace of $\mathbb{R}^n$. Any linearly independent set of exactly p elements in H is automatically a basis for H. Also, any set of p elements of H that spans H is automatically a basis for H.

Let $v_1, v_2, \dots, v_p \in H$

lin ind means $A = \begin{bmatrix} v_1 & v_2 & \cdots & v_p \end{bmatrix}$ has no free vbls...

Claim ① If $b \in H$ then $b = Ax$ for some $x \in \mathbb{R}^p$

trying to show there is x so that

$= \underbrace{v_1 x_1 + v_2 x_2 + \cdots + v_p x_p}_{}$

Span of the vectors $v_1, v_2, \dots, v_p$ ...

Idea... $[A|b]$ show that I can solve this ...

Hypothesis: H is a p-dim subspace of $\mathbb{R}^n$

means H has a basis with p vectors...

So there are vectors $w_1, w_2, \ldots, w_p$ that form a basis for H. That is

① $w_1, w_2, \ldots, w_p$ span H

② $w_1, w_2, \ldots, w_p$ lin. independent

Let $M = [w_1 | w_2 | \cdots | w_p]$

Note since M is a basis then $b \in H$ implies there is $y \in \mathbb{R}^p$ such that $b = My$...

Since $v_1, v_2, \ldots, v_p \in H$ then

$$v_1 = My_1, \quad v_2 = My_2, \ldots, \quad v_p = My_p$$

for some vectors $y_1, y_2, \ldots, y_p \in \mathbb{R}^p$

15 **The Basis Theorem**

Let H be a p-dimensional subspace of $\mathbb{R}^n$. Any linearly independent set of exactly p elements in H is automatically a basis for H. Also, any set of p elements of H that spans H is automatically a basis for H.

We'll work an example of the Basis Theorem on Friday and then explain how the above ideas fit together to make a proof of that theorem..