Math 330

Section 1004
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$$A = \begin{bmatrix} 2 & -3 \\ 5 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -2 \\ 5 & 3 \end{bmatrix}$$

$$(f \circ g)(x,y) = f(g(x,y)) = f(1x-2y,\ 5x+3y)$$

$$= \begin{bmatrix} 2(1x-2y) - 3(5x+3y) \\ 5(1x-2y) + 4(5x+3y) \end{bmatrix} = \begin{bmatrix} \boxed{-13x} & -13y \\ 25x & +2y \end{bmatrix}$$

$$C = \begin{bmatrix} -13 & -13 \\ 25 & 2 \end{bmatrix}$$

How to multiply matrices

$$B = \begin{bmatrix} 1 & -2 \\ 5 & 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -3 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} -13 & -13 \\ 25 & \boxed{2} \end{bmatrix}$$

answer $C = AB$

linear Algebra is all about linear functions:

$$f(x,y) = (2x - 3y,\ 5x + 4y)$$

$$A = \begin{bmatrix} 2 & -3 \\ 5 & 4 \end{bmatrix}$$

what is the identity function?

$$id(x,y) = (x,y)$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$id(x,y) = \begin{bmatrix} 1x + 0y \\ 0x + 1y \end{bmatrix}$$

Some simple kinds of linear functions that aren't quite as simple as the identity...

ELEMENTARY ROW OPERATIONS $r_i \leftarrow r_i - \alpha r_j \quad i \neq i$

1. (Replacement) Replace one row by the sum of itself and a multiple of another row.[1]
2. (Interchange) Interchange two rows. $r_i \longleftrightarrow r_j \quad i \neq j$
3. (Scaling) Multiply all entries in a row by a nonzero constant.

$r_i \leftarrow \alpha r_i \quad \alpha \neq 0$

It'd be nice to know what matrices correspond to these linear functions...

Row operations

Examples

1. $r_i \leftarrow r_i - \alpha r_j$

$$A = \begin{bmatrix} 2 & -3 \\ 5 & 4 \end{bmatrix} \qquad r_2 \leftarrow r_2 - \frac{5}{2} r_1$$

result

$$U = \begin{bmatrix} 2 & -3 \\ 0 & \frac{23}{2} \end{bmatrix}$$

what matrix corresponds to this linear function

$r_2 \leftarrow r_2 + \frac{5}{2} r_1$

new result

$$\begin{bmatrix} 2 & -3 \\ 5 & 4 \end{bmatrix}$$

linear function on the left

Eliminate x:

mult $5/2$

$$\begin{cases} 2x - 3y = 7 \\ 5x + 4y = 3 \end{cases}$$

$$5x - \frac{15}{2} y = \frac{35}{2}$$

$$\frac{23}{2} y = \frac{-29}{2}$$

transformed the system $\Rightarrow$

$$\begin{cases} 2x - 3y = 7 \\ \frac{23}{2} y = \frac{-29}{2} \end{cases}$$

$r_2 \leftarrow r_2 + \frac{5}{2} r_1$

trick to figure out what matrix corresponds to any linear function is to perform that linear function on the identity matrix...

$$(f \circ id)(x,y) = f(x,y)$$

substitute matrix I here

over here I get the matrix for f...

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad r_2 \leftarrow r_2 + \frac{5}{2} r_1 \quad \begin{bmatrix} 1 & 0 \\ \frac{5}{2} & 1 \end{bmatrix} = L$$

I've got three matrices....

$A = \begin{bmatrix} 2 & -3 \\ 5 & 4 \end{bmatrix}$ $U = \begin{bmatrix} 2 & -3 \\ 0 & \frac{23}{2} \end{bmatrix}$ $L = \begin{bmatrix} 1 & 0 \\ \frac{5}{2} & 1 \end{bmatrix}$

Claim: $A = LU$.

$\frac{5}{2}(-3) + 1 \cdot \frac{23}{2} = \frac{-15+23}{2} = \frac{8}{2} = 4$

$U = \begin{bmatrix} 2 & -3 \\ 0 & \frac{23}{2} \end{bmatrix}$

answer $= LU = A$

$L = \begin{bmatrix} 1 & 0 \\ \frac{5}{2} & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 5 & 4 \end{bmatrix}$

Why is this useful?

The original problem:

$$\boxed{\begin{aligned} 2x - 3y &= 7 \\ 5x + 4y &= 3 \end{aligned}}$$

$f(x,y) = (2x-3y,\ 5x+4y)$

original problem in different notation...

wanted to solve $\boxed{f(x,y) = (7,3) \text{ for } x \text{ and } y}$

$L = \begin{bmatrix} 1 & 0 \\ \frac{5}{2} & 1 \end{bmatrix}$ $l(x,y) = (x,\ \frac{5}{2}x + y)$

$u = \begin{bmatrix} 2 & -3 \\ 0 & \frac{23}{2} \end{bmatrix}$ $u(x,y) = (2x-3y,\ \frac{23}{2}y)$

$$f(x,y) = \ell(u(x,y)) = (7,3)$$

Thus $\quad u(x,y) = \ell^{-1}(7,3)$

since ℓ corresponds to a lower triangular matrix, solving for $u(x,y)$ above is easy...

similarly $\quad (x,y) = u^{-1}(\ell^{-1}(7,3))$ . .

In practice solve for (a,b) such that

by substitution

$$\ell(a,b) = (7,3) \qquad \begin{cases} a = 7 \\ \frac{5}{2}a + b = 3 \end{cases}$$

then solve for (x,y) such that

$$u(x,y) = (a,b) \qquad \begin{cases} 2x - 3y = a \\ \frac{13}{2}y = b \end{cases}$$