## Augmented Matrix:

$$1x_1 - 7x_2 + 0x_3 + 6x_4 = 5$$
$$0x_1 + 0x_2 + 1x_3 - 2x_4 = -3$$
$$-1x_1 + 7x_2 - 4x_3 + 2x_4 = 7$$

12. $\left[\begin{array}{cccc|c} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{array}\right]$

$r_3 \leftarrow r_3 + r_1$ $\left[\begin{array}{cccc|c} \boxed{1} & -7 & 0 & 6 & 5 \\ 0 & 0 & \boxed{1} & -2 & -3 \\ 0 & 0 & -4 & 8 & 12 \end{array}\right]$

$r_3 \leftarrow r_3 + 4r_2$ $\begin{array}{c} \begin{array}{cccc} P & F & P & F \end{array} \\ \left[\begin{array}{cccc|c} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$ ← row of zeros means inf. many soln...

left side of the system — right side of the system

What are pivots: The first non-zero entry in each row after making it triangular.

I did the elimination steps needed to make a triangular matrix (echelon form) and by luck ended up with the reduced echelon form..

by luck, in this case otherwise more work.

- ☑ The pivots are the only non-zero entries in their columns ⑤ ← in the book Chap 1.2
- ☑ Also the pivots need to be scaled so they are exactly equal 1. ④

After finding reduced echelon form, find soln:

$$\begin{array}{cccc|c} P & F & P & F & \\ \end{array}$$

$$\left[\begin{array}{cccc|c} \boxed{1} & -7 & 0 & 6 & 5 \\ 0 & 0 & \boxed{1} & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} \boxed{x_1} - 7x_2 + 6x_4 = \boxed{5} \\ \boxed{x_3} - 2x_4 = \boxed{-3} \end{cases}$$

$x_3 = -3 + 2x_4$

$x_1 = 5 + 7x_2 - 6x_4$

write solution in vector form

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 5 + 7x_2 - 6x_4 \\ x_2 \\ -3 + 2x_4 \\ x_4 \end{bmatrix}$$

factor out the constant and the coeficients in front of $x_2$ & $x_4$.

$$= \begin{bmatrix} 5 \\ 0 \\ -3 \\ 0 \end{bmatrix} + \boxed{\begin{bmatrix} 7 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -6 \\ 0 \\ 2 \\ 1 \end{bmatrix} x_4}$$

constant vector

free variable

free variables

this part is called the null space.

Came from the right hand side of the system

came from the left hand side...

all linear combinations of the vectors $\begin{bmatrix} 7 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -6 \\ 0 \\ 2 \\ 1 \end{bmatrix}$

This is a linear function of $x_2$ and $x_4$

$$\begin{bmatrix} 7 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -6 \\ 0 \\ 2 \\ 1 \end{bmatrix} x_4 = n(x_2, x_4)$$

$$n(x_2, x_4) = \begin{bmatrix} 7x_2 - 6x_4 \\ x_2 \\ 2x_4 \\ x_4 \end{bmatrix} \quad \& \quad \text{matrix is } N = \begin{bmatrix} 7 & -6 \\ 1 & 0 \\ 0 & 2 \\ 0 & 1 \end{bmatrix}$$

Solution to the problem:

$$\boxed{x = \begin{bmatrix} 5 \\ 0 \\ -3 \\ 0 \end{bmatrix} + N \begin{bmatrix} x_2 \\ x_4 \end{bmatrix}}$$

2 dimension space (a set that includes the zero vector)

2 dimension set of solutions.

That product

$$\begin{bmatrix} x_2 \\ x_4 \end{bmatrix}$$

$$N = \begin{bmatrix} 7 & -6 \\ 1 & 0 \\ 0 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 7x_2 - 6x_4 \\ x_2 \\ 2x_4 \\ x_4 \end{bmatrix}$$

check work...

$$\begin{array}{r} 9 \\ -25 \\ \hline -16 \end{array} \qquad \begin{array}{r} 25 \\ 9 \\ \hline 16 \end{array}$$

→ One more from section 1.2

**4.** $\begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{bmatrix}$

$r_2 \leftarrow r_2 - 3r_1$

$r_3 \leftarrow r_3 - 5r_1$

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -16 & -34 \end{bmatrix}$$

$r_3 \leftarrow r_3 - 2r_2$

$$\begin{bmatrix} \boxed{1} & 3 & 5 & 7 \\ 0 & \boxed{-4} & -8 & -12 \\ 0 & 0 & 0 & \boxed{-10} \end{bmatrix}$$

echelon form

U

$r_1 \leftarrow r_1 + \frac{3}{4} r_2$

$$\begin{bmatrix} \boxed{1} & 0 & -1 & -2 \\ 0 & \boxed{-4} & -8 & -12 \\ 0 & 0 & 0 & \boxed{-10} \end{bmatrix}$$

$r_1 \leftarrow r_1 - \frac{2}{10} r_3$

$r_2 \leftarrow r_2 - \frac{12}{10} r_3$

$$\begin{bmatrix} \boxed{1} & 0 & -1 & 0 \\ 0 & \boxed{-4} & -8 & 0 \\ 0 & 0 & 0 & \boxed{-10} \end{bmatrix}$$

rescale the pivots to make them 1.

$r_2 \leftarrow \frac{1}{-4} r_2$

$r_3 \leftarrow \frac{1}{-10} r_3$

$$R = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

reduced row-echelon form