A rectangular matrix is in **echelon form** (or **row echelon form**) if it has the following three properties:

1. All nonzero rows are above any rows of all zeros.
2. Each leading entry of a row is in a column to the right of the leading entry of the row above it.
3. All entries in a column below a leading entry are zeros.

If a matrix in echelon form satisfies the following additional conditions, then it is in **reduced echelon form** (or **reduced row echelon form**):

4. The leading entry in each nonzero row is 1.
5. Each leading 1 is the only nonzero entry in its column.

3. $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix}$

$r_2 \leftarrow r_2 - 4r_1$

$r_3 \leftarrow r_3 - 6r_1$

$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -9 \\ 0 & -5 & -10 & -15 \end{bmatrix}$

$r_3 \leftarrow r_3 - \frac{5}{3} r_2$

$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -9 \\ 0 & 0 & 0 & 0 \end{bmatrix} = U$

Echelon form of the orig. matrix.

rescale so the pivots are all 1's.

$r_2 \leftarrow -\frac{1}{3} r_2$

need to eliminate this two.

$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$r_1 \leftarrow r_1 - 2r_2$

$\begin{array}{cccc} P & P & F & F \end{array}$

$\begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ reduced eschelon form....

The pivot columns are 1 and 2.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix} = LU$$

5 Describe the possible

Factor...

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 6 & 5/3 & 1 \end{bmatrix} \begin{bmatrix} \boxed{1} & 2 & 3 & 4 \\ 0 & \boxed{-3} & -6 & -9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

To undo these row operations

$r_2 \leftarrow r_2 - 4r_1$

$r_3 \leftarrow r_3 - 6r_1$

$r_3 \leftarrow r_3 - \frac{5}{3} r_2$

Undo them in reverse order one at a time

$r_3 \leftarrow r_3 + \frac{5}{3} r_2$

$r_3 \leftarrow r_3 + 6r_1$

$r_2 \leftarrow r_2 + 4r_1$

row col.

in the 3,2 slot place 5/3

Augmented matrix

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 4 & 5 & 6 & 7 & 0 \\ 6 & 7 & 8 & 9 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} \boxed{1} & 0 & -1 & -2 & 0 \\ 0 & \boxed{1} & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

reduced row echelon form

Solving the system

$$\begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 = 0 \\ 4x_1 + 5x_2 + 6x_3 + 7x_4 = 0 \\ 6x_1 + 7x_2 + 8x_3 + 9x_4 = 0 \end{cases}$$

$$\begin{array}{c} \\ \\ R= \end{array}\begin{array}{c} \begin{array}{ccccc} P & P & F & F & \\ \end{array} \\ \underbrace{\left[\begin{array}{cccc|c} \boxed{1} & 0 & -1 & -2 & 0 \\ 0 & \boxed{1} & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]} \end{array}$$

rrm

zero or null vector

$$x_1 - x_3 - 2x_4 = 0$$
$$x_2 + 2x_3 + 3x_4 = 0$$
$$0 = 0$$

short cut answer

set of solutions

$$\left\{ \begin{array}{l} x_2 = -2x_3 - 3x_4 \\ x_1 = x_3 + 2x_4 \end{array} \right.$$

skip this...

In vector form:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_3 + 2x_4 \\ -2x_3 - 3x_4 \\ x_3 \\ x_4 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} 2 \\ -3 \\ 0 \\ 1 \end{bmatrix} x_4}$$

linear combination of vectors... set of all such combinations is called a subspace...

Called the nullspace of A.

Can write this as matrix mult.

In matrix form:

$$x = \overbrace{\begin{bmatrix} 1 & 2 \\ -2 & -3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}}^{N} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} = N \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}$$

nullspace matrix

Check

$$\begin{bmatrix} x_3 \\ x_4 \end{bmatrix}$$

answer

$$\begin{bmatrix} 1 & 2 \\ -2 & -3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_3 + 2x_4 \\ -2x_3 - 3x_4 \\ x_3 \\ x_4 \end{bmatrix}$$

What we looked at during class from the textbook and margin notes:

1.3 VECTOR EQUATIONS

Importar

$$\begin{bmatrix} 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 1+2 \\ -2+5 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

Linear Combinations $\boxed{\checkmark}$

DEFINITION

If $\mathbf{v}_1, \dots, \mathbf{v}_p$ are in $\mathbb{R}^n$, then the set of all linear combinations of $\mathbf{v}_1, \dots, \mathbf{v}_p$ is denoted by Span $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ and is called the **subset of** $\mathbb{R}^n$ **spanned** (or **generated**) **by** $\mathbf{v}_1, \dots, \mathbf{v}_p$. That is, Span $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ is the collection of all vectors that can be written in the form

subspace

$$c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_p\mathbf{v}_p$$

with $c_1, \dots, c_p$ scalars.

The section is mostly about notation...

$$S = \begin{bmatrix} v_1 & v_2 & \cdots & v_p \end{bmatrix}$$

Span $(v_1, \dots, v_p)$

$$= \left\{ S \begin{bmatrix} c_1 \\ \vdots \\ c_p \end{bmatrix} : \text{for all choices of } c_i \right\}$$

set builder notation

$$\{x : x > 0\} = (0, \infty)$$

what does

$x \in \mathbb{R}$

mean?

Here is some more notation we will use...

what about

$x \in \mathbb{R}^4$

means $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ where $x_1 \in \mathbb{R}$, $x_2 \in \mathbb{R}$, $x_3 \in \mathbb{R}$, $x_4 \in \mathbb{R}$

what about

$A \in \mathbb{R}^{3 \times 4}$

matrix with 3 rows
4 columns
and entries that are real numbers.