$$f(x) = Ax = \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} 4 & 0 & 5 \\ -1 & 3 & 2 \end{bmatrix} \end{array} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4x_1 + 0x_2 + 5x_3 \\ -x_1 + 3x_2 + 2x_3 \end{bmatrix}$$

$$g(x) = Bx = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 7 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 + x_3 \\ 3x_1 + 5x_2 + 7x_7 \end{bmatrix}$$

What does it mean to add two matrices? Add the corresponding linear functions...

$$(f+g)(x) = f(x) + g(x)$$

$$= \begin{bmatrix} 4x_1 + 0x_2 + 5x_3 \\ -x_1 + 3x_2 + 2x_3 \end{bmatrix} + \begin{bmatrix} x_1 + x_2 + x_3 \\ 3x_1 + 5x_2 + 7x_7 \end{bmatrix}$$

$$= \begin{bmatrix} 4x_1 + 0x_2 + 5x_3 + x_1 + x_2 + x_3 \\ -x_1 + 3x_2 + 2x_3 + 3x_1 + 5x_2 + 7x_7 \end{bmatrix}$$

$$= \begin{bmatrix} 5x_1 + x_2 + 6x_3 \\ 2x_1 + 8x_2 + 9x_3 \end{bmatrix} = \begin{bmatrix} 5 & 1 & 6 \\ 2 & 8 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Chapter 2.1

$$\begin{array}{ccccc} A & & B & & AB \\ \begin{bmatrix} * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{bmatrix} & & \begin{bmatrix} * & * \\ * & * \\ * & * \\ * & * \\ * & * \end{bmatrix} & = & \begin{bmatrix} * & * \\ * & * \\ * & * \end{bmatrix} \\ 3 \times 5 & & 5 \times 2 & & 3 \times 2 \end{array}$$

input of A

output of B

Match

Size of AB

What is 3^2? $3 \cdot 3$

What is $3^{1/2}$? $\sqrt{3}$

what is $3^{p/q}$? $\sqrt[q]{3^p}$

What is $3^{\sqrt{2}}$?

let $\frac{p_n}{q_n}$ be a sequence of fractions

that converges to $\sqrt{2}$.

Define $3^{\sqrt{2}} = \lim_{n \to \infty} 3^{p_n/q_n}$

Let $A \in \mathbb{R}^{n \times n}$ then A^k makes sense

$A^0 = I$, $A^k = \underbrace{A \cdots A}_{k}$

With this definition it makes sense to plug A into a polynomial function

$$p(x) = 3x^4 + 2x^3 - 5x^2 + 7x - 18$$

$$p(A) = 3A^4 + 2A^3 - 5A^2 + 7A - 18I_{n \times n}$$

What about $\sin A$? Use Taylor polynomial for sin function, plug A into the Taylor Poly and take limits

Transposition of a matrix or vector.

switch rows and columns...

$$x = \begin{bmatrix} 1 \\ 2 \\ 4 \\ -1 \end{bmatrix} \qquad y = \begin{bmatrix} 2 \\ 3 \\ -1 \\ 2 \end{bmatrix}$$

$$x \cdot y = 1 \cdot 2 + 2 \cdot 3 + 4 \cdot (-1) + (-1)(2)$$

$$x^T = \begin{bmatrix} 1 & 2 & 4 & -1 \end{bmatrix}$$

$$x^T y = \begin{bmatrix} 1 & 2 & 4 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ -1 \\ 2 \end{bmatrix} = 1 \cdot 2 + 2 \cdot 3 + 4 \cdot (-1) + (-1)(2)$$

Let A and B denote matrices whose sizes are appropriate for the following sums and products.

a. $(A^T)^T = A$

b. $(A+B)^T = A^T + B^T$

c. For any scalar r, $(rA)^T = rA^T$

d. $\underbrace{(AB)^T}_{C^T} = \underbrace{B^T A^T}_{D}$

most interesting...

$$A_{2\times \cancel{3}} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \qquad B_{\cancel{3}\times 4} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \end{bmatrix}$$

$AB = C$

$(AB)^T = C^T$

$$\underset{3\times 4}{\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \end{bmatrix}}$$

$$\underset{2\times 3}{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}} \underset{2\times 4}{\begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \end{bmatrix}}$$

How are the c's related to the a's and the b's?

$$c_{11} = a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31}$$

$$\vdots$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33}$$

In general

$$\boxed{c_{ij} = \sum_k a_{ik} b_{kj}}$$

$$C^T = \begin{bmatrix} c_{11} & c_{21} \\ c_{12} & c_{22} \\ c_{13} & c_{23} \\ c_{14} & c_{24} \end{bmatrix}$$

The entry of C^T in the ith row and j-th column

$$[C^T]_{ij} = c_{ji}$$

Recall

$$A_{2\times \cancel{3}} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \qquad B_{\cancel{3}\times 4} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \end{bmatrix}$$

3×4

$$A^T = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{bmatrix} \qquad B^T = \begin{bmatrix} b_{11} & b_{21} & b_{31} \\ b_{12} & b_{22} & b_{32} \\ b_{13} & b_{23} & b_{33} \\ b_{14} & b_{24} & b_{34} \end{bmatrix}$$

3×2 $\qquad$ 4×3

$$B^T A^T = D$$

$4\times\cancel{3}\ \cancel{3}\times 2 \quad 4\times 2$

$$\begin{bmatrix} b_{11} & b_{21} & b_{31} \\ b_{12} & b_{22} & b_{32} \\ b_{13} & b_{23} & b_{33} \\ b_{14} & b_{24} & b_{34} \end{bmatrix} \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \\ d_{31} & d_{32} \\ d_{41} & d_{42} \end{bmatrix}$$

Question is $D = C^T$?

$$C^T = \begin{bmatrix} c_{11} & c_{21} \\ c_{12} & c_{22} \\ c_{13} & c_{23} \\ c_{14} & c_{24} \end{bmatrix}$$

$$d_{11} = b_{11}a_{11} + b_{21}a_{12} + b_{31}a_{13}$$

$$c_{11} = a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31}$$

$$d_{32} = \underbrace{b_{13}a_{21}} + \underbrace{b_{23}a_{22}} + \underbrace{b_{33}a_{23}}$$

$$c_{23} = a_{21}\underbrace{b_{13}} + a_{22}\underbrace{b_{23}} + a_{23}\underbrace{b_{33}}$$

In general compare these formula...

$$d_{ij} = \sum_k b_{ki}a_{jk} \xrightarrow{\text{plug in}} c_{ij} = \sum_k a_{ik}b_{kj}$$

$$[C^T]_{ij} = c_{ji} = \sum_k a_{jk}b_{ki}$$

(The same)