Section 1005
Jan 20, '22

Multiplication of Matrices:

$$\begin{cases} 2x - 3y = 1 \\ 4x - 5y = 5 \end{cases}$$

starting point ... systems of linear equation...

$f : \mathbb{R}^2 \to \mathbb{R}^2$ ← all 2-vectors with real elements

$f(x,y) = (2x-3y,\ 4x-5y)$

inputs related to the columns

$$f(x,y) = \begin{bmatrix} 2x - 3y \\ 4x - 5y \end{bmatrix} \quad \xleftrightarrow{\text{corresponding between}} \quad A = \begin{array}{cc} x \text{ coef.} & y \text{ coef} \\ \downarrow & \downarrow \end{array}\begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix} \begin{array}{l} \leftarrow \text{eq 1} \\ \leftarrow \text{eq 2} \end{array}$$

outputs related to the rows

$g : \mathbb{R}^2 \to \mathbb{R}^2$

linear functions and matrices

$$g(x,y) = \begin{bmatrix} -3x + 2y \\ 5x - 4y \end{bmatrix} \quad \longleftrightarrow \quad B = \begin{bmatrix} -3 & 2 \\ 5 & -4 \end{bmatrix}$$

$$f(1,2) = (2\cdot1 - 3\cdot2,\ 4\cdot1 - 5\cdot2) = (-4, -6)$$

means the same thing as

$$\begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -4 \\ -6 \end{bmatrix}$$

Graphically how to organize your work

$$\begin{array}{cc} & \begin{bmatrix} 1 \\ 2 \end{bmatrix} \\ \begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix} & \begin{bmatrix} -4 \\ -6 \end{bmatrix} \end{array}$$

answer...

$2\cdot1 + (-3)(2) =$

$$f(x,y) = (2x-3y,\ 4x-5y)$$

$$g(x,y) = (-3x+2y,\ 5x-4y)$$

$$(f\circ g)(x,y) = f(g(x,y)) = f(-3x+2y,\ 5x-4y)$$

$$= \big(2(-3x+2y) - 3(5x-4y),\ 4(-3x+2y) - 5(5x-4y)\big)$$

$$= \Big(\big(2\cdot(-3) + (-3)(5)\big)x + \big(2\cdot 2 + (-3)\cdot(-4)\big)y,\ \big(4(-3)+(-5)(5)\big)x + \big(4\cdot 2 + (-5)(-4)\big)y\Big)$$

$$= (-21x + 16y,\ -37x + 28y)$$

By definition

$$C = AB$$

→ matrix corresponding to $f \circ g$

$$C = \begin{bmatrix} -21 & 16 \\ -37 & 28 \end{bmatrix}$$

Organize matrix-matrix multiplication like this....

$$B = \begin{bmatrix} -3 & 2 \\ 5 & -4 \end{bmatrix}$$

answer

$$A = \begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} -21 & 16 \\ -37 & 28 \end{bmatrix}$$

Don't add fractions like this

$$\cancel{\frac{1}{3} + \frac{1}{2} = \frac{2}{5}}$$

Don't mult matrices like this

$$\cancel{\begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 5 & -4 \end{bmatrix} = \begin{bmatrix} -6 & -6 \\ 20 & 20 \end{bmatrix}}$$

System of linear equations:

$$\begin{cases} 2x - 3y = 1 \\ 4x - 5y = 5 \end{cases} \qquad A = \begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix}$$

Gaussian Elimination is just eliminating things in a careful order.

Idea take first eq. mult it by 2 and subtract it from the second equation...

$$\begin{array}{r} \cancel{4x} - 5y = 5 \\ \cancel{4x} - 6y = 2 \\ \hline y = 3 \end{array}$$

Rather than substituting y back in ...

$$\begin{cases} 2x - 3y = 1 \\ y = 3 \end{cases}$$

~~2~~ now these two equations

$$U = \begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$$

Elementary row operations:

1. $r_i \leftarrow r_i - \alpha r_j \qquad i \neq j$
2. $r_i \leftrightarrow r_j$ where $i \neq j$
3. $r_i \leftarrow \alpha r_i$ where $\alpha \neq 0$

ELEMENTARY ROW OPERATIONS

1. (Replacement) Replace one row by the sum of itself and a multiple of another row.[1] $r_i \leftarrow r_i - \alpha r_j$
2. (Interchange) Interchange two rows. $r_i \leftrightarrow r_j$
3. (Scaling) Multiply all entries in a row by a nonzero constant. $r_i \leftarrow \alpha r_i$

$$A = \begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix} \quad r_2 \leftarrow r_2 - 2r_1$$

$$\begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$$

identity$(x, y) \rightarrow (x, y)$

matrix for this function...

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Remark

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \end{bmatrix} \qquad \begin{bmatrix} 5 \\ 7 \end{bmatrix} \leftarrow \text{answer}$$

Thus $I \begin{bmatrix} 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$

• What function performs $r_2 \leftarrow r_2 - 2r_1$?

Idea compose this function with the identity to write it in matrix form.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$r_2 \leftarrow r_2 - 2r_1$

$$\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$e(x,y) = (x, -2x+y)$

This is the function that performs to row operation $r_2 \leftarrow r_2 - 2r_1$

$$\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$$

means $r_2 \leftarrow r_2 - 2r_1$

undo this operation how

Check...

$$\begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix}$$

answer

$$\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$$

$$r_2 \leftarrow r_2 - 2r_1$$

and then

$$r_2 \leftarrow r_2 + 2r_1$$

I'm back to where I started....

$$\begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$$

$$A = L \quad U$$