LU factorization....

Section 1005
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$$\begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$$

$$A = L \quad U$$

Check this factorization...

same

$$U = \begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$$

answer...

$$L = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix}$$

What to do with the factorization:

$$\ell(x,y) = (x, 2x+y) \qquad L = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$u(x,y) = (2x-3y, y) \qquad U = \begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$$

$$f(x,y) = (2x-3y, 4x-5y) = \ell(u(x,y))$$

I want to solve

$$f(x,y) = (1,5)$$

this is the same as

$$\underbrace{l(u(x,y))}_{} = (1,5)$$

since f is factored like this I can now find the answer is two steps...

In other words, undoing to composition

$$l(\text{something}) = (1,5)$$

$$u(x,y) = \text{something}.$$

since l and u are triangular these are simple problems...

$$l(a,b) = (1,5)$$

$$u(x,y) = (a,b)$$

Solve these two systems to find the answer to the original

$$l(a,b) = (a, 2a+b) = (1,5)$$

or

$$\begin{cases} a = 1 \\ 2a + b = 5 \end{cases}$$

1

by substitution

$$b = 5 - 2\cdot 1 = 3$$

$$u(x,y) = (2x - 3y, y) = (a,b)$$

or

$$\begin{cases} 2x - 3y = a = 1 \\ y = b = 3 \end{cases}$$

2

$$x = \frac{1 + 3\cdot 3}{2} = 5$$

ANSWER $(x,y) = (5,3)$

Supposed to solve $\begin{cases} 2x - 3y = 1 \\ 4x - 5y = 5 \end{cases}$

plug in $(x,y) = (5,3)$ to check:

$2\cdot 5 - 3\cdot 3 = 1$
$4\cdot 5 - 5\cdot 3 = 5$

same, so it's the correct answer...

about any problem at all..

TWO FUNDAMENTAL QUESTIONS ~~ABOUT A LINEAR SYSTEM~~

1. Is the system consistent; that is, does at least one solution *exist*?
2. If a solution exists, is it the *only* one; that is, is the solution *unique*?

~~well~~ posedness of a mathematics problem

there is exactly one solution

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\begin{cases} x + 2y + 3z = 1 \\ 4x + 5y + 6z = 2 \\ 7x + 8y + 9z = 3 \end{cases}$$

Try to factor the matrix using row operations...

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \begin{matrix} r_2 \leftarrow r_2 - 4r_1 \checkmark \\ r_3 \leftarrow r_3 - 7r_1 \checkmark \end{matrix} \quad \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{bmatrix} \quad r_3 \leftarrow r_3 - 2r_2 \checkmark$$

So 2 goes in the (3,2) slot of L

$$u = \begin{array}{c} \begin{array}{ccc} P & P & F \end{array} \\ \begin{bmatrix} \boxed{1} & 2 & 3 \\ 0 & \boxed{-3} & -6 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

eschelon form for the matrix A

$$A = L \quad U$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{array}{c|c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 7 & 2 & 1 \end{bmatrix} \end{array} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

If you forget how to take the row operations and plop them into the right places to make L it's difficult... but if you remember then it's easy...

$$A = LU$$

$$\begin{cases} a = 1 \\ 4a + b = 2 \\ 7a + 2b + c = 3 \end{cases} \quad \text{then} \quad \begin{cases} x + 2y + 3z = a \\ -3y - 6z = b \\ 0 = c \end{cases}$$

Only has a solution when $c = 0$, but it's not... or is it?