Math 430 Quiz 1 Version A

1. We say V is the direct sum of subspaces $U_1, \ldots, U_m$, written $V = U_1 \oplus \cdots \oplus U_m$, if

(A) for each $j \neq k$ then $U_k \cap U_j = \{0\}$.

(B) for each $u_j \in U_j$ and $u_k \in U_k$ with $j \neq k$ then u_j is orthogonal to u_k.

(C) each $v \in V$ can be written uniquely as a sum $u_1 + \cdots + u_m$, where $u_j \in U_j$.

(D) $U_1 \cup \cdots \cup U_m$ is a subspace.

(E) none of these.

2. Suppose U_1, U_2 are subspaces of V. The sum $U_1 + U_2$ is

(A) the largest subspace of V containing $U_1 \cap U_2$.

(B) the smallest subspace of V containing $U_1 \cap U_2$.

(C) the largest subspace of V containing $U_1 \cup U_2$.

(D) the smallest subspace of V containing $U_1 \cup U_2$.

(E) none of these.

3. Let $(v_1, \ldots, v_m)$ be a list of vectors in V. Then

(A) $\operatorname{span}(v_1, \ldots, v_m) = \{ a_1 v_1 + \cdots + a_m v_m : a_1, \ldots, a_m \in V \}$.

(B) $\operatorname{span}(v_1, \ldots, v_m) = \{ a_1 v_1 + \cdots + a_m v_m : a_1, \ldots, a_m \in \mathbf{F} \}$.

(C) $\operatorname{span}(v_1, \ldots, v_m) = \{ a_1 v_1 \cup \cdots \cup a_m v_m : a_1, \ldots, a_m \in V \}$.

(D) $\operatorname{span}(v_1, \ldots, v_m) = \{ a_1 v_1 \cup \cdots \cup a_m v_m : a_1, \ldots, a_m \in \mathbf{F} \}$.

(E) none of these.

4. A vector space V is called finite dimensional

(A) if some finite list $(v_1, \ldots, v_m)$ of vectors spans V.

(B) if the cardinality of any subspace of V is finite.

(C) if every combination of vectors in V is orthogonal.

(D) if there are subspaces $U_1, \ldots, U_m$ such that $V = U_1 \oplus \cdots \oplus U_m$.

(E) none of these.

5. A list of vectors $(v_1, \ldots, v_m)$ in V are linearly dependent if and only if

(A) every choice of $a_1, \ldots, a_m \in \mathbf{F}$ makes $a_1 v_1 + \cdots + a_m v_m$ equal 0.

(B) the only choice of $a_1, \ldots, a_m \in \mathbf{F}$ that makes $a_1 v_1 + \cdots + a_m v_m$ equal 0 is $a_1 = \cdots = a_m = 0$.

(C) there exists $a_1, \ldots, a_m \in \mathbf{F}$, not all 0, such that $a_1 v_1 + \cdots + a_m v_m = 0$.

(D) there exists $a_1, \ldots, a_m \in \mathbf{F}$, not all 0, such that $a_1 v_1 + \cdots + a_m v_m \neq 0$.

(E) none of these.

6. Use

> **Theorem 2.6:** In a finite-dimensional vector space, the length of every linearly independent list of vectors is less than or equal to the length of every spanning list of vectors.

to prove

> **Theorem 2.14:** Any two bases of a finite-dimensional vector space have the same length.

7. Prove one of the following:

Proposition 1.8: Suppose that $U_1, \ldots, U_n$ are subspaces of V. Then $V = U_1 \oplus \cdots \oplus U_n$ if and only if both the following conditions hold:

(a) $V = U_1 + \cdots + U_n$;

(b) the only way to write 0 as a sum $u_1 + \cdots + u_n$, where each $u_j \in U_j$, is by taking all the u_j's equal to 0.

Proposition 2.8: A list $(v_1, \ldots, v_n)$ of vectors in V is a basis of V if and only if every $v \in V$ can be written uniquely in the form

$$v = a_1 v_1 + \cdots + a_n v_n,$$

where $a_1, \ldots, a_n \in \mathbf{F}$.