Math 430 Quiz 2 Version A

1. For $T \in \mathcal{L}(V, W)$ the null space of T is defined as

(A) $\text{null}(T) = \{ w \in W : Tw = 0 \}$.

(B) $\text{null}(T) = \{ v \in V : Tv = 0 \}$.

(C) $\text{null}(T) = \{ Tw : w \in W \}$.

(D) $\text{null}(T) = \{ Tv : v \in V \}$.

(E) none of these.

2. For $T \in \mathcal{L}(V, W)$ the range of T is defined as

(A) $\text{range}(T) = \{ w \in W : Tw = 0 \}$.

(B) $\text{range}(T) = \{ v \in V : Tv = 0 \}$.

(C) $\text{range}(T) = \{ Tw : w \in W \}$.

(D) $\text{range}(T) = \{ Tv : v \in V \}$.

(E) none of these.

3. Let $T \in \mathcal{L}(V, W)$ and $(v_1, \ldots, v_n)$ be a basis of V and $(w_1, \ldots, w_m)$ be a basis of W. With respect to these basis, the matrix $M(T)$ is defined as the $m \times n$ matrix with entries $a_{j,k}$ where

(A) $Tv_k = a_{1,k}w_1 + \cdots + a_{m,k}w_m$.

(B) $Tv_j = a_{j,1}w_1 + \cdots + a_{j,m}w_m$.

(C) $w_k = a_{1,k}Tv_1 + \cdots + a_{n,k}Tv_n$.

(D) $w_j = a_{j,1}Tv_1 + \cdots + a_{j,n}Tv_n$.

(E) none of these.

4. Let $T \in \mathcal{L}(V)$ and U be a subspace of V. We say that U is invariant under T if

(A) for every $Tu \in U$ then $u \in U$.

(B) for every $v \notin U$ then $Tv \notin U$.

(C) $T|_U \in \mathcal{L}(V)$.

(D) $T|_U \in \mathcal{L}(U)$.

(E) none of these.

5. A scaler $\lambda \in \mathbf{F}$ is called an eigenvalue of $T \in \mathcal{L}(V)$ if

(A) there exists a vector $u \in V$ such that $Tu = \lambda u$.

(B) there exists a nonzero vector $u \in V$ such that $Tu = \lambda u$.

(C) the operator $T + \lambda I$ is not injective.

(D) the operator $T + \lambda I$ is not surjective.

(E) none of these.

6. Work one of the following homework problems:

> **§3#5:** Suppose that $T \in \mathcal{L}(V, W)$ is injective and $(v_1, \ldots, v_n)$ is linearly independent in V. Prove that $(Tv_1, \ldots, Tv_n)$ is linearly independent in W.
>
> **§3#7:** Prove that if $(v_1, \ldots, v_n)$ spans V and $T \in \mathcal{L}(V, W)$ is surjective, then $(Tv_1, \ldots, Tv_n)$ spans W.

7. Prove one of the following:

> **Theorem 5.6:** Let $T \in \mathcal{L}(V)$. Suppose $\lambda_1, \ldots, \lambda_m$ are distinct eigenvalue of T and $v_1, \ldots, v_m$ are corresponding nonzero eigenvectors. Then $(v_1, \ldots, v_n)$ is linearly independent.
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> **Theorem 5.10:** Every operator on a finite-dimensional, nonzero, complex vector space has an eigenvalue.