Math 430/630 Quiz 3 Version A

1. A list of vectors is called orthonormal if and only if

(A) the vectors in it are pairwise orthogonal and each vector has norm 1.
(B) for every $v \in V$ there exists $k \in \mathbf{N}$ such that $\langle v, e_k \rangle = 0$.
(C) for every $v \in V$ there exists $k \in \mathbf{N}$ such that $\langle v, e_k \rangle = 1$.
(D) for every $v \in V$ there exists $k \in \mathbf{N}$ such that $\|v\| = \|e_k\|$.
(E) none of these.

2. An orthonormal basis of V is

(A) any list of linearly independent vectors $(e_1, \ldots, e_n)$ that span V.
(B) equal to range(Q) where $Q \in \mathcal{L}(V)$ is an orthogonal operator.
(C) obtained by setting $e_i = v_i/\|v_i\|$ where $(v_1, \ldots, v_n)$ is a basis of V.
(D) an orthonormal list of vectors in V that is also a basis of V.
(E) none of these.

3. The orthogonal complement of U, denoted $U^\perp$, is given by

(A) $U^\perp = \{\, v \in V : \text{ there exists } u \in U \text{ such that } \langle v, u \rangle = 0 \,\}$.
(B) $U^\perp = \{\, v \in V : \text{ there exists } u \in U \text{ such that } \langle v, u \rangle = 1 \,\}$.
(C) $U^\perp = \{\, v \in V : \langle v, u \rangle = 0 \text{ for all } u \in U \,\}$.
(D) $U^\perp = \{\, v \in V : \langle v, u \rangle = 1 \text{ for all } u \in U \,\}$.
(E) none of these.

4. Let P be the orthogonal projection of V onto U. Given $v \in V$ let $v = u + w$ where $u \in U$ and $w \in U^\perp$. Then generally

(A) $Pv = w$.
(B) $Pu = v$.
(C) $Pv = u$.
(D) $Pu = w$.
(E) none of these.

5. Let V and W be finite-dimensional complex inner product spaces and $T \in \mathcal{L}(V, W)$. The adjoint of T, denoted T^*, is the unique linear map in $\mathcal{L}(W, V)$ such that

(A) $\langle Tv, w \rangle = \langle T^*w, v \rangle$ for every $v \in V$ and $w \in W$.
(B) $\langle Tv, w \rangle = \langle v, T^*w \rangle$ for every $v \in V$ and $w \in W$.
(C) $TT^*w = w$ for every $w \in W$.
(D) $T^*Tv = v$ for every $v \in V$.
(E) none of these.

6. Let

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 5 \end{bmatrix}.$$

Find an orthogonal matrix Q and an upper triangular matrix R such that $A = QR$.

7. Prove one of following:

Corollary 6.33: If U is a subspace of V, then $U = (U^{\perp})^{\perp}$.

Adjoint of ajoint: If $T \in \mathcal{L}(V, W)$, then $T = (T^*)^*$.

8. Extra Credit: Prove

Triangle Inequality: If $u, v \in V$ then $\|u + v\| \leq \|u\| + \|v\|$.