Math 430/630 Quiz 4 Version A

Note, unless otherwise stated, that $\mathbf{F}$ denotes $\mathbf{R}$ or $\mathbf{C}$ and V is a finite-dimensional, nonzero, inner-product space over $\mathbf{F}$.

1. An operator $T \in \mathcal{L}(V)$ is called self-adjoint if and only if

(A) $T^* = T$.

(B) $T^* = T^{-1}$.

(C) $T^*T = TT^*$.

(D) $\|Tv\| = \|T^*v\|$ for all $v \in V$.

(E) none of these.

2. An operator on an inner-product space is called normal if and only if

(A) T has a p.d.f. of the form $\dfrac{1}{\sigma\sqrt{2\pi}} \exp\Big(-\dfrac{(x-\mu)^2}{2\sigma^2}\Big)$.

(B) T has a p.d.f. of the form $\dfrac{1}{\sigma^2 2\pi} \exp\Big(-\dfrac{(x-\mu)^2}{2\sigma^2}\Big)$.

(C) $\|Tv\| = \|v\|$ for every $v \in V$.

(D) $T^*T = TT^*$.

(E) none of these.

3. An operator $T \in \mathcal{L}(V)$ is called positive if and only if

(A) T is normal and $\langle Tv, v\rangle \geq 0$ for all $v \in V$.

(B) T is normal and $\langle Tv, w\rangle \geq 0$ for all $v, w \in V$.

(C) T is self-adjoint and $\langle Tv, v\rangle \geq 0$ for all $v \in V$.

(D) T is self-adjoint and $\langle Tv, w\rangle \geq 0$ for all $v, w \in V$.

(E) none of these.

4. An operator $S \in \mathcal{L}(V)$ is called a square root of an operator $T \in \mathcal{L}(V)$ if and only if

(A) S is positive and $S^2 = T$.

(B) S is self-adjoint and $S^2 = T$.

(C) $S^2 = T$.

(D) $S^*S = SS^* = T$.

(E) none of these.

5. An operator $S \in \mathcal{L}(V)$ is called an isometry if and only if

(A) for every $v \in V$ there exists $\theta \in \mathbf{R}$ such that $Sv = e^{i\theta}v$.

(B) $\|Sv\| = \|v\|$ for every $v \in V$.

(C) $S^*S = SS^*$.

(D) $S^* = S$.

(E) none of these.

6. Prove one of the following:

Proposition 7.2: Let V be a finite dimensional, nonzero, complex inner-product space. If T is an operator on V such that $\langle Tv, v\rangle = 0$ for all $v \in V$, then $T = 0$.

Proposition 7.4: Let V be a finite dimensional, nonzero, real inner-product space. If T is a self-adjoint operator on V such that $\langle Tv, v\rangle = 0$ for all $v \in V$, then $T = 0$.

Proposition 7.6: Let V be a finite dimensional, nonzero, inner-product space over $\mathbf{F}$. An operator $T \in \mathcal{L}(V)$ is normal if and only if $\|Tv\| = \|T^*v\|$ for all $v \in V$.

7. Extra Credit: Clearly state both the complex spectral theorem and the real spectral theorem. Compare and contrast the two theorems. Be as specific as possible.