Machine Epsilon

Your work should be presented in the form of a typed report using clear and properly punctuated English. Where appropriate include full program listings and output. If you choose to work in a group of two, please turn in independently prepared reports.

1a. Given $x \in \mathbf{R}$ let x^* denote the floating point approximation of x. Write a program to find the greatest δ_1 such that

$$(x^* + \delta_1)^* = x^*.$$

You may use the bisection method with initial interval of $[0, x^*]$. Run your program for x^* ranging over the set $\{1, 2, 3, 4, 5, 10, 100, 1000\}$ using single precision floating point and again using double precision.

b. For $x \in \mathbf{R}$ fixed define

$$\delta_2 = \min\{\, \delta : (x^* + \delta)^* = x^* \,\}.$$

Write a program to determine whether $\delta_1 = -\delta_2$. What happens theoretically when chopping is used for x^*? How about when rounding is used?

c. Let J be a bounded subset of $\mathbf{R}$. Define $J^* = \{\, x^* : x \in J \,\}$ and $I = \{\, x : x^* \in J^* \,\}$. The maximum relative error

$$\epsilon = \max_{x \in I} \frac{|x - x^*|}{|x|} \geq \max_{x \in I} \frac{|x - x^*|}{|x^*| + |x - x^*|} = \max_{x^* \in J^*} \frac{\delta}{|x^*| + \delta}$$

where $\delta = \max\{\, \delta_1, -\delta_2 \,\}$. Since relative error is bounded by 5×10^{-n} where n is the number of significant digits, then

$$n \leq \log_{10} \left\{ 5 \min_{x^* \in J^*} \left(\frac{|x^*| + \delta}{\delta} \right) \right\}.$$

Write a program to compute this upper bound for n using both single and double precision floating point. Take the set

$$J^* = \{\, 1, 2, 3, 4, 5, 10, 100, 1000 \,\}.$$