Cholesky Factorization

1a. Write a program to find the Cholesky factorization $A = U^T U$ of a symmetric positive definite 2×2 matrix A. Your program should report an error if the input matrix is not symmetric or not positive definite; otherwise it should report the factor U.

b. Test your program using the matrices

$$\begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix},$$

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}, \quad \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, \quad \begin{bmatrix} 5 & -1 \\ -1 & 3 \end{bmatrix}.$$

Which ones are symmetric positive definite? For those that are, what is the factor U?

c. The file `prog2c.dat` contains square matrices of varying sizes separated by blank lines. The description of each matrix consists of an integer specifying the dimension of the matrix followed by the entries of the matrix in row-major order. Use the Matlab command `chol` to check which of these matrices are positive definite and to find the Cholesky decomposition for each one that is.

d. Go to the Guide for Available Mathematical Software at

$$\texttt{http://math.nist.gov/}$$

and click on

- What problem it solves.

Search for `Cholesky` and locate a subroutine for the Cholesky factorization of a real symmetric positive definite matrix. Download this subroutine and print it off.

e. [Math/CS 666 and Extra Credit]. Write a program that uses the Cholesky routines from `LAPACK`, `CLAPACK`, `LAPACK++` or similar to determine whether the matrices in the file `prog2c.dat` are positive definite. Either install the entire `LAPACK` library or directly use `DPOTF2` for simplicity. Hint: For these routines work you will also need to have BLAS, the Basic Linear Algebra Subprogram library, installed. You may use the reference BLAS implementation at Netlib or the automatically tuned implementation called ATLAS.