1. The MATLAB computer codes

```
function r=qeq1(a,b,c)
    if b<0
        r1=(-b+sqrt(b^2-4*a*c))/(2*a);
        r2=(2*c)/(-b+sqrt(b^2-4*a*c));
    else
        r1=(2*c)/(-b-sqrt(b^2-4*a*c));
        r2=(-b-sqrt(b^2-4*a*c))/(2*a);
    end
    r=[r1,r2];
```

and

```
function r=qeq2(a,b,c)
    if b>0
        r1=(-b+sqrt(b^2-4*a*c))/(2*a);
        r2=(2*c)/(-b+sqrt(b^2-4*a*c));
    else
        r1=(2*c)/(-b-sqrt(b^2-4*a*c));
        r2=(-b-sqrt(b^2-4*a*c))/(2*a);
    end
    r=[r1,r2];
```

both compute the roots of the quadratic equation $ax^2 + bx + c = 0$.

(i) Which routine will result in a computation with less rounding error?

(A) The routine `qeq1`.

(B) The routine `qeq2`.

(C) There is no difference.

(ii) Explain your answer to the above question.

2. Consider the table of divided differences for the function $f(x) = e^x$ given by

x_n	$f(x_n)$	$f(x_n, x_{n+1})$	$f(x_n, \ldots, x_{n+2})$	$f(x_n, \ldots, x_{n+3})$
0.0	1.00000	1.10700	0.63480	0.244688
0.2	1.22140	1.42440	0.83055	0.314238
0.5	1.64872	1.92273	1.08194	
0.8	2.22554	2.46370		
1.0	2.71828			

Find the interpolating polynomial passing through the points

$$(0.2, 1.22140),\ (0.5, 1.64872),\ (0.8, 2.22554),\ (1.0, 2.71828).$$

3. Find the Taylor polynomial of degree 4 for the function $f(x) = 1/(2 + x)$ expanded around $x_0 = 0$.

4. Fill in the blanks in the following statement of Theorem 4.2.1 on the error in polynomimal interpolation.

Theorem 4.2.1. Let $n \geq 0$, let $f(x)$ have $n+1$ continuous derivatives on $[a, b]$, and let $x_0, x_1, \ldots, x_n$ be distinct node points in $[a, b]$. Let $p_n(x)$ be the unique interpolating polynomial of degree less than or equal n passing through the points $(x_k, f(x_k))$ for $k = 0, 1, \ldots, n$. Then

$$f(x) - p_n(x) = \boxed{}$$

for $a \leq x \leq b$, where c_x is an unknown point between the minimum and maximum of $x_0, x_1, \ldots, x_n$, and x.

5. Consider the interpolating polynomial $p_4(x)$ of degree 4 passing through the points $(x, \exp(x))$ where $x = 0.0, 0.2, 0.5, 0.8, 1.0$. Let

$$M = \max_{x \in [0,1]} |x(x-0.2)(x-0.5)(x-0.8)(x-1)| \approx 0.0026735.$$

Use Theorem 4.2.1 and the value of M given above to find a bound for $\exp(x) - p_4(x)$ on the interval $[0, 1]$.

6. Bound the error in the approximation $\cos(x) = 1 - \frac{1}{2}x^2$ for $-0.2 \leq x \leq 0.2$.

7. Given the true value x_T with an approximation x_A define the following:

(i) The absolute error $\text{Error}(x_A)$

(ii) The relative error $\text{Rel}(x_A)$

8. Let $x_A = 0.08$ be an approximation of x_T. If $|\text{Rel}(x_A)| \le 0.05$ what is the largest number x_T could have been?

9. Let x_A and y_A be approximations of x_T and y_T with absolute errors $\text{Error}(x_A) = 0.03$ and $\text{Error}(y_A) = 0.04$. Assuming exact arithemetic, what is $\text{Error}(x_A + y_A)$?

10. Write pseudocode for the following:

(i) Newton's method to find an approximation x_A with $|\text{Error}(x_A)| < \epsilon$ of a root of $f(x)$ with the starting guess x_0.

(ii) The secant method to find an approximation x_A with $|\text{Error}(x_A)| < \epsilon$ of a root of $f(x)$ with the starting guesses x_0 and x_1.

11. Compare Newton's method to the secant method method and state the advantages and disadvantages of each method.