Math/CS 466/666 Fall 2008 Exam 2

1. Write psuedocode to efficiently evaluate the polynomial

$$p(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$$

given a value of x and an array a_i of coefficients as inputs.

2. Consider using the trapeziod method and Simpson's method to approximate

$$\int_1^2 \frac{1}{t}\, dt$$

with $h = (2-1)/n$ where $n = 20$. Without actually performing the computation, tell which method will yeild a better approximation? Explain why in as mathematically precise way as possible?

3. The nodal points x_i and the weights w_i for the Gauss quadrature methods with $n = 2$, 3 and 4 are given in the table

n	x_i	w_i
2	± 0.5773502692	1.0
3	± 0.7745966692	0.5555555556
	0.0	0.8888888889
4	± 0.8611363116	0.3478548451
	± 0.3399810436	0.6521451549

Make the substitution $x = 2t - 3$ to rewrite the integral

$$\int_1^2 \frac{1}{t}\,dt \quad \text{in the form} \quad \int_{-1}^1 f(x)\,dx$$

and then use the Gauss quadrature method with $n = 3$ to approximate this integral.

4. The Gauss quadrature formula with $n = 7$ is exact for all polynomials of degree less than or equal at most

(A) 7

(B) 13

(C) 14

(D) 27

(E) none of these.

5. Use Taylor's theorem to estimate the mathematical error in the approximation

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}.$$

6. Explain how roundoff error implies a minimum optimal size for h and how many significant digits are lost by the subtraction of two nearly equal numbers in the numerator of the above formula when performing numerical differentiation.

7. Let f be a differentiable function defined on $\mathbf{R}$ with root α. Given an initial guess of x_0 state Newton's method for approximating α.

(i) Suppose f is twice continuously differentiable and $f'(\alpha) \neq 0$. Show that Newton's method is quadratically convergent.

(ii) Suppose that α is a root of multiplicity 3 so that $f'(\alpha) = 0$ and $f''(\alpha) = 0$. In this case the quotients

$$\lambda_n = \frac{x_{n+1} - x_n}{x_n - x_{n-1}}$$

will converge to

(A) 1/2
(B) 2/3
(C) 1/3
(D) 3/2
(E) none of these.