MATH/CS 466/666 FALL 2008 QUIZ 2

1. Calculate the error and relative error in the approximation $x_A \approx x_T$ where $x_T = 0.028254$ and $x_A = 0.028271$.

2. For $x \in (-1, 1)$ the functions defined by

$$f(x) = (1+x)^{1/3} - 1$$

and

$$g(x) = \frac{x}{\big((1+x)^{1/3}+1\big)(1+x)^{1/3}+1}$$

are mathematically equivalent. If x is very close to zero, which function will evaluate more accuratly on a digital computer?

(A) $f(x)$

(B) $g(x)$

(C) There is no difference.

3. Let $x_A = 0.06$ by an approximation of x_T. If $|\mathrm{Error}(\mathrm{x_A})| \leq 0.003$ what is largest number that x_T could have been?

4. Let x_A and y_A be approximations of x_T and y_T with relative errors $\text{Rel}(x_A) = 0.03$ and $\text{Rel}(y_A) = 0.04$. Assuming exact arithemetic, what is $\text{Rel}(x_A y_A)$?

5. The computer codes

```
s=0.0
for n from 1 to 1000
do
      s=s+1.0/n
end
```

and

```
s=0.0
for n from 1 to 1000
do
      s=s+1.0/(1001-n)
end
```

both computes the sum $\sum_{n=1}^{1000} \frac{1}{n}$ as `s`. Which one computes `s` more accurately?

(A) The first code..

(B) The second code.

(C) There is no difference.

6. Suppose f is continuously differentiable, $f(\alpha) = 0$ and $f'(\alpha) \neq 0$.

(i) Newton's method for approximating α given an initial guess x_0 is

(A) $x_{n+1} = x_n + f(x_n)/f'(x_n)$

(B) $x_{n+1} = x_n - f(x_n)/f'(x_n)$

(C) $x_{n+1} = x_n + f'(x_n)/f(x_n)$

(D) $x_{n+1} = x_n - f'(x_n)/f(x_n)$

(E) none of these

(ii) Show Newton's method converges quadratically in a neighborhood of α.

7. Compare Newton's method to the bisection method.

(i) State the advantages and disadvantages of each method.

(ii) Give an example where the bisection method would be preferred.

(iii) Give and example where Newton's method would be preferred.