Math/CS 466/666 Fall 2008 Quiz 4

1. Let A be a invertible $n \times n$ matrix. Define $\text{cond}(A)$ the condition number of A.

2. Let x_a be an approximation of the solution x to $Ax = b$ where A is an $n \times n$ matrix and b is a vector of length n. Define $r = b - Ax_a$. Show that

$$\frac{\|x - x_a\|}{\|x\|} \le \text{cond}(A)\frac{\|r\|}{\|b\|}.$$

3. Give a simple formula for the sum $\displaystyle\sum_{k=1}^{n-1} k^2$.

4. Let A and B be $n \times n$ matrices with entries a_{ij} and b_{ij} respectively. Define $C = AB$. The standard way of computing the elements c_{ij} of C is

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}.$$

How many multiplications does it take to fully compute C in this way?

5. Let A be an $n \times n$ matrix that can be written as $A = LU$ where L is lower triangular and U is upper triangular. Explain in details the total number of multiplications and divisions generally needed to find L and U using Gauss-Jordan elimination.

6. The nodal points x_i and the weights w_i for the Gauss quadrature methods with $n = 2$, 3 and 4 are given in the table

n	x_i	w_i
2	± 0.5773502692	1.0
3	± 0.7745966692	0.5555555556
	0.0	0.8888888889
4	± 0.8611363116	0.3478548451
	± 0.3399810436	0.6521451549

Make the substitution $x = (t - 3)/2$ to rewrite the integral

$$\int_1^5 \log(t)\,dt \quad \text{in the form} \quad \int_{-1}^1 f(x)\,dx$$

and then use the Gauss quadrature method with $n = 3$ to approximate this integral.

7. The Gauss quadrature formula with $n = 4$ is exact for all polynomials of degree less than or equal at most

(A) 7

(B) 13

(C) 14

(D) 27

(E) none of these.