Exam I Review

Tue Oct 7 10:44:12 PDT 2008 Version 1

Please know the following for the exam Friday, Oct 10.

1. **Be able to do all problems from the quizes and homework.**
   a. Note that some homework problems are too lengthy or computational for an exam question. Please understand these homework problems anyway, as I may simplify them for the exam.
2. Taylor's Theorem.
   a. Know how to create the Taylor polynomial.
   b. Know the remainder term.
   c. Know how to use the remainder term to estimate errors.
3. Polynomial evaluation.
   a. Know how to use nested multiplication and synthetic division.
   b. How many multiplications and additions are required to evaluate a typical n-th degree polynomial?
4. IEEE Single and Double Precision floating point.
   a. How many bytes?
   b. How many significant digits?
   c. Which fractions can be represented exactly and which can not.
5. Definition of absolute error $\text{Error}(x_A)$ and relative error $\text{Rel}(x_A)$.
6. Types of Errors: Modeling errors, physical measurement errors, machine representation and arithmetic errors and mathematical approximation errors.
7. The difference between chopping and rounding modes with the advantages and disadvantages of each.
8. Propagated error in multiplication and addition.

$$\text{Rel}(x_A y_A) = \text{Rel}(x_A) + \text{Rel}(y_A) - \text{Rel}(x_A)\text{Rel}(y_A)$$

and

$$\text{Error}(x_A + y_A) = \text{Error}(x_A) + \text{Error}(y_A).$$

9. Propageted error in function evaluation.

$$\text{Error}(f(x_A)) = f'(x)\text{Error}(x_A)$$

and

$$\text{Rel}(f(x_A)) = \frac{f'(x)}{f(x)}\text{Rel}(x_A).$$

10. How to sum a series of terms to minimize rounding error.
11. How to rearrange certain algebraic expressions, for example, the quadratic formula, to minimize loss of precision due to subtraction of two nearly equal numbers.
12. Root Finding.
    a. State the bisection method, Newton's method and the secant method.
    b. Compare the advantages and disadvantages of each of these three methods.
    c. Show that Newton's method is quadratically convergent.
    d. How to use Newton's method to find roots of multiplicity.
13. Polynomial Interpolation.
    a. Definition of the Lagrange basis functions.
    b. Definition of divided differences.
    c. Statement of Theorem 4.2.1 for the error in polynomial interpolation.
14. Understand and be able to reproduce the examples from the book:
    a. Example 1.2.4
    b. Page 40 loop example for x=x+0.1 and problems 4, 5 on page 42.
    c. Loss of precision example on page 48.
    d. Taylor series example on page 49.
    e. Example 3.2.2
    f. Given $x_0, \ldots, x_n$ and $f(x_0), \ldots, f(x_n)$ be able to compute divided differences $f(x_0, x_1)$, $f(x_1, x_2)$, $f(x_0, x_1, x_2)$ and so forth. Use these divided differences in formulae (4.33)–(4.35) on page 129 to find interpolating polynomials.