

**Do not use the computer in any way for this part of the quiz. Also keep your book and notes closed.**

1. State Taylor's theorem with remainder for a scalar function in one variable.

2. State the trapezoid method for computing a definite integral.

3. Show that that a real symmetric matrix must have real eigenvalues.

4. Let  $A = \begin{bmatrix} 1 & 3 & 6 \\ -4 & -1 & -2 \\ 0 & 1 & -1 \end{bmatrix}$ .

(i) Find  $\|A\|_\infty$

(ii) Find  $\|A\|_1$

5. Prove that Newton's method is quadratically convergent. You may assume that the initial guess  $x_0$  is close to the root  $c$  of  $f$  and that  $f'(c) \neq 0$ .

6. Solve one of the following:

(i) Prove that if  $A$  is diagonally dominant then Jacobi's iterative method for solving  $Ax = b$  converges.

(ii) Suppose  $Ax = b$  and let  $x^*$  be an approximation of  $x$  with residual  $r = b - Ax^*$ . Show that

$$\frac{\|x - x^*\|}{\|x\|} \leq \|A\| \|A^{-1}\| \frac{\|r\|}{\|b\|}.$$

**Please use the Ubuntu VM for this part of the quiz. You may also use your notes and textbooks as well as online resources such as Wikipedia and Google. However, do not use email or any other messaging service.**

Submit your program and output using the commands

```
/nfs/home/ejolson/opt/bin/submit -q1 program.c  
/nfs/home/ejolson/opt/bin/submit -q2 output.txt
```

Here `program.c` is the name of your program and `output.txt` is an output file obtained by running the program with the command

```
./a.out >output.txt
```

If you wish to change any part of your submission simply retype the appropriate submit command again. You may check each of your submissions with the command

```
/nfs/home/ejolson/opt/bin/submit -pn
```

where `n` is equal the number used in submit command.

7. Please answer one of the following questions.

- (i) Write or modify a C computer program to use Euler's explicit method for approximating  $y(2)$  where  $y(t)$  solves the differential equation  $y' = \cos(t + y^2)$  with initial condition  $y(0) = 1$ . Run your program with  $n = 256$  times steps of size  $h = 2/n$  and print the resulting value of  $y(2)$ .
- (ii) Write or modify a C computer program to use the power method to approximate the eigenvalue with the largest magnitude for the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 0 & 3 \end{bmatrix}.$$

Take  $x^0 = (1, 0, 0)$  and print the first 10 iterations of the method.

8. [Math/CS 666 and Extra Credit] Please solve the other problem appearing above and submit your answer using the commands

```
/nfs/home/ejolson/opt/bin/submit -q3 program2.c  
/nfs/home/ejolson/opt/bin/submit -q4 output2.txt
```