**Do not use the computer in any way for this part of the quiz. Also keep your book and notes closed.**

**1.** Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

**(i)** Find $\|A\|_1$

**(ii)** Find $\|A\|_\infty$

**(iii)** Find $B = A^\dagger A$ where $A^\dagger$ denotes the conjugate transpose of $A$.

**(iv)** The matrix $B$ defined above has the eigenvalues $\sigma(B) = \{2, 3, 6\}$. Find $\|A\|_2$.

**2.** State Taylor's theorem with remainder for a scalar function in one variable.

**3.** Use Taylor's theorem to show there exists $c$ between $x-h$ and $x+h$ such that

$$\frac{f(x+h)-f(x-h)}{2h}=f'(x)+\frac{f'''(c)}{6}h^2.$$

**4.** Use Taylor's theorem to show there exists $c$ between $x-h$ and $x+h$ such that

$$\frac{f(x+h)-f(x)+f(x-h)}{h^2}=f''(x)+\frac{f''''(c)}{12}h^2.$$

**5.** State Jacobi's iterative method for solving $Ax = b$.

**6.** State the RK4 tableau for the classical fourth-order Runge-Kutta method.

**7.** State the power method for approximating the spectral radius of a matrix $A$.

**8.** Answer one of the following:

**(i)** Prove that strict diagonal dominance of $A$ implies that $A$ is invertible.

**(ii)** Find the region of absolute stability for the Euler explicit method applied to the problem $u' = \alpha u$. Then do the same for the trapezoid method.

**9.** Answer one of the following:

**(i)** Define Newton's method and prove that it is quadratically convergence. Assume the initial guess $x_0$ is close to the root $c$ of $f$ and that $f'(c) \neq 0$.

**(ii)** Suppose $Ax = b$ and let $x^*$ be an approximation to $x$ with residual $r = b - Ax^*$. Define $\operatorname{cond}(A)$ and show that

$$\frac{\|x - x^*\|}{\|x\|} \leq \operatorname{cond}(A) \frac{\|r\|}{\|b\|}.$$

**Please use the Ubuntu VM for this part of the quiz. You may also use your notes and textbooks as well as online resources such as Wikipedia and Google. However, do not use email or any other messaging service.**

**10.** Answer two of the following questions:

**(i)** Write or modify a C computer program to use the power method to compute the largest eigenvalue to 5 significant digits for the matrix $B = A^\dagger A$ where $A^\dagger$ denotes the conjugate transpose of $A$ and

$$A = \begin{bmatrix} -6 & -6 & -1 & 7 \\ 3 & 6 & -7 & 8 \\ -2 & 9 & 8 & -1 \\ -5 & -8 & -2 & 8 \end{bmatrix}.$$

**(ii)** Write or modify a C computer program to use Gram–Schmidt orthogonalization to factor the matrix $A$ appearing in problem (i) above as $QR$ where $Q$ is orthogonal and $R$ is upper triagular.

**(iii)** Write or modify a C computer program to use Simpson's quadrature to approximate the definite integral

$$\int_0^2 \frac{1}{x^3+1}\,dx$$

to 5 significant digits.

**(iv)** Write or modify a C computer program to implement the Runge–Kutta third-order method given by the tableau

$$\begin{array}{c|ccc} 0 & & & \\ \frac{1}{2} & \frac{1}{2} & & \\ 1 & -1 & 2 & \\ \hline & \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \end{array}$$

to approximate $y(3)$ to 5 significant digits where $y(t)$ satisfies

$$y' = \sin(ty), \qquad y(0) = 1.$$

Submit your answers using the following commands:

```
/nfs/home/ejolson/opt/bin/submit -q1 program1.c
/nfs/home/ejolson/opt/bin/submit -q2 output1.txt
/nfs/home/ejolson/opt/bin/submit -q3 program2.c
/nfs/home/ejolson/opt/bin/submit -q4 output2.txt
```

Please include a comment at the top of each submitted program source-code file to indicate which question it answers.

**11.** [Extra Credit] Consider the partial differential equation

$$\frac{\partial y}{\partial t} = 0.02\, y'' + 0.3\, xy' - 0.01\, y$$

with boundary and initial conditions

$$y(t,0) = 0, \quad y(t,3) = \sin 5t, \quad y(0,x) = 0.$$

where the primes indicate partial differentiation with respect to $x$. Write or modify a C program to approximate the value of $y(4, 1.5)$ to 5 decimal places.

Submit the extra credit using the commands

```
/nfs/home/ejolson/opt/bin/submit -q5 program3.c
/nfs/home/ejolson/opt/bin/submit -q6 output3.txt
```