Math/CS 466/666 Final Part 2 Version A

Instructions: Create a subdirectory called `final` and copy the files `p1.c`, `p2.c` and `p3.c` from the website. Each of these files contains an incomplete program and your goal is to complete each program so that it correctly performs each of the calculations further described below. After you are finished please place the output of each of your programs into the file `p1.out`, `p2.out` and `p3.out` respectively. If you have any trouble doing this please ask for help. Finally, archive the directory containing all your work and submit the archive file using the commands

```
$ cd ..
$ tar cf final.tar final
$ submit final.tar
```

If you change any program and wish to resubmit your answers please repeat both the `tar` and `submit` commands above. Before leaving, please check with me at the front of the room to ensure that your submission is complete. Feel free to ask for help if you have any trouble with the submit program. Following is a detailed description of the calculation each program is supposed to perform:

1. Add the missing code in the subroutines `f`, `ft` and `fu` of the computer program `p1.c` so that it solves the initial value problem

$$u' = e^{-u}\sin(t+u) \qquad \text{with} \qquad u(0)=1$$

on the interval $[0,10]$ using 1024 time steps of Taylor's second-order method.

2. Consider the third-order approximation $f''(x) \approx S_h(f)$ where

$$S_h(f) = \frac{35f(x) - 104f(x+\frac{1}{2}h) + 114f(x+h) - 56f(x+\frac{3}{2}h) + 11f(x+2h)}{3h^2}.$$

Fill in the missing code for the routine `Rh` to provide a fourth-order approximation $R_h(f)$ of $f''(x)$ using Richarson extrapolation of the form

$$R_h(f) = c_1 S_h(f) + c_2 S_{2h}(f)$$

with a suitable choice of constants c_1 and c_2 so the resulting program `p2.c` prints a table showing h, $R_h(f)$ and $|R_h(f) - f'(x)|$ when $f(x) = 1/(x^2 - x + 3)$, $x = 1$ and $h = 2^{-n}$ for $n = 0, 1, \ldots, 7$.

3. Add the missing code in the definition of `w[K]` in the computer program `p3.c` so that it approximates the integral $\int_0^3 e^{-x^2}\,dx$ by subdividing the interval $[0,3]$ into eight subintervals, rescaling each subinterval to $[-1,1]$, applying the quadrature formula

$$\int_{-1}^{1} f(x)dx \approx \sum_{k=1}^{2} w_k f(x_k)$$

where $x = (-5/8, 5/8)$ and then summing the results over all the subintervals. Make sure your choice of weights w_k has been chosen such that

$$\int_{-1}^{1} x^j\,dx = \sum_{k=1}^{2} w_k x_k^j \qquad \text{is exact for} \qquad j = 0, 1.$$