Math/CS 466/666 Quiz 2 Version A

Instructions: Create a subdirectory called `quiz2` and copy the files `p1.c`, `p2.c` and `p3.c` from the website. Each of these files contains an incomplete program and your goal is to complete each program so that it correctly performs each of the calculations further described below. After you are finished please place the output of each of your programs into the file `p1.out`, `p2.out` and `p3.out` respectively. If you have any trouble doing this please ask for help. Finally, archive the directory containing all your work and submit the archive file using the commands

```
$ cd ..
$ tar cf quiz2.tar quiz2
$ submit quiz2.tar
```

Before leaving, please check with me at the front of the room to ensure that your submission is complete. Feel free to ask for help if you have any trouble with the submit program. Following is a detailed description of the calculation each program is supposed to perform:

1. Add the missing code in the functions `f`, `ft` and `fu` of the computer program `p1.c` so that it solves the initial value problem

$$u' = u^2 \cos t \qquad \text{with} \qquad u(1) = 2$$

on the interval $[1, 6]$ using 1024 time steps of Taylor's second-order method.

2. Add the missing code in the definition of `w[K]` in the computer program `p2.c` so that it approximates the integral $\int_0^3 e^{-x^2} dx$ by subdividing the interval $[0, 3]$ into eight subintervals, rescaling each subinterval to $[-1, 1]$, applying the quadrature formula

$$\int_{-1}^{1} f(x)dx \approx \sum_{k=1}^{4} w_k f(x_k)$$

where $x = (-3/4, -1/4, 1/4, 3/4)$ and then summing the results over all the subintervals. Make sure your choice of weights w_k has been chosen such that

$$\int_{-1}^{1} x^j dx = \sum_{k=1}^{4} w_k x_k^j \qquad \text{is exact for} \qquad j = 0, 1, 2, 3.$$

3. Add the missing code in the function `g` of the computer program `p3.c` so that it performs five iterations of Newton's method to approximate $\sqrt{5}$ starting with an initial guess of $x_0 = 1$.

4. [Extra Credit] It is known that

$$e = \lim_{n\to\infty} x_n \qquad \text{where} \qquad x_n = \left(1 + \frac{1}{n}\right)^n.$$

Write a program to compute $E_n = |x_n - e|$ and make a table showing the values of n and E_n for $n = 1, \ldots, 20$. Plot E_n versus n using log-log coordinates and determine numerically the rate at which $E_n \to 0$ as $n \to \infty$. Include in the comments of your program a description of the plotting commands needed to process the output and the rate of convergence that you found.