Math/CS 466/666 Exam Part 2 Version A

Instructions: Create a subdirectory called `exam` and copy the files `p1.c`, `p2.c` and `p3.c` from the website. Each of these files contains an incomplete program and your goal is to complete each program so that it correctly performs each of the calculations further described below. After you are finished please place the output of each of your programs into the file `p1.out`, `p2.out` and `p3.out` respectively. If you have any trouble doing this please ask for help. Finally, archive the directory containing all your work and submit the archive file using the commands

```
$ cd ..
$ tar cf exam.tar exam
$ submit exam.tar
```

If you change any program and wish to resubmit your answers please repeat both the `tar` and `submit` commands above. Before leaving, please check with me at the front of the room to ensure that your submission is complete. Feel free to ask for help if you have any trouble with the submit program. Following is a detailed description of the calculation each program is supposed to perform:

1. Add the missing code in the subroutine `rk4` of the computer program `p1.c` so that it solves the initial value problem

$$u' = e^{-u}\sin(t+u) \qquad \text{with} \qquad u(0) = 1$$

on the interval $[0, 10]$ using 1024 time steps of the RK4 method.

2. Add the missing code to the `main` routine in the computer program `p2.c` so that it solved the system of linear equations given by $Ax = b$ where

$$A = \begin{bmatrix} -3 & 1 & -6 & 5 & -5 \\ 0 & -5 & -6 & 3 & 9 \\ -6 & -6 & 6 & 8 & -6 \\ 9 & -3 & -5 & 2 & -4 \\ -6 & 8 & -7 & -9 & -8 \end{bmatrix} \qquad \text{and} \qquad b = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$

by using Gaussian elimination with partial pivoting to factor $A = PLU$ where P is a permutation matrix, L is a lower-triangular matrix with 1's on the diagonal and U is an upper triagular matrix. Then forward and back substituting to find x.

3. Consider the first-order approximation of the derivative given by

$$f'(x) \approx D_h(f) \qquad \text{where} \qquad D_h(f) = (f(x+h) - f(x))/h.$$

Fill in the missing code for the routine `Rh` to provide a second-order approximation $R_h(f)$ using Richarson extrapolation of the form

$$R_h(f) = c_1 D_h(f) + c_2 D_{2h}(f)$$

with a suitable choice of constants c_1 and c_2 so the resulting program `p3.c` prints a table showing h, $R_h(f)$ and $|R_h(f) - f'(x)|$ when $f(x) = 1/(x^2 - x + 3)$, $x = 1$ and $h = 2^{-n}$ for $n = 0, 1, \ldots, 7$.