How to compute $\|A\|_1$ and $\|A\|_\infty$.

$$\|A\|_1 = \max\left\{ \sum_{i=1}^{n} |A_{ik}| : k=1,\dots,n \right\}.$$

and

$$\|A\|_\infty = \max\left\{ \sum_{j=1}^{n} |A_{kj}| : k=1,\dots,n \right\}$$

we proved the formula for $\|A\|_1$ in class, the formula for $\|A\|_\infty$ can be found using similar techiques.

Here is an example using these formulas for

$$A = \begin{bmatrix} -6 & 5 & -3 \\ 2 & -1 & -2 \\ 4 & 7 & 1 \end{bmatrix}$$

Computing $\|A\|_\infty$ is maximums of sums of absolute values along rows

$$A = \begin{bmatrix} -6 & 5 & -3 \\ 2 & -1 & -2 \\ 4 & 7 & 1 \end{bmatrix}$$

$6+5+3 = 14$

$2+1+2 = 5$

$4+7+1 = 12$

Largest is 14 so...

$\|A\|_\infty = 14$

Next compute $\|A\|_1$ which is the maximum of sums of absolute values along colums

$$A = \begin{bmatrix} -6 & 5 & -3 \\ 2 & -1 & -2 \\ 4 & 7 & 1 \end{bmatrix}$$

$3+2+1 = 6$

$5+1+7 = 13$

$6+2+4 = 12$

Largest is 13

so

$\|A\|_1 = 13$

The Lagrange polynomial basis functions are defined

$$l_j(t) = \prod_{i \neq j} \frac{t - x_i}{x_j - x_i}$$

and the polynomial passing through the points (x_j, y_j) for $j = 1, \dots, n$ is given by

$$p(t) = \sum_{j=1}^{n} y_j \, l_j(t)$$

Here is an example to find the polynomial passing through the points

$(1,3)$ $(4,2)$ and $(-2,8)$

First

$$l_1(t) = \frac{(t-4)(t+2)}{(1-4)(1+2)} = \frac{(t-4)(t+2)}{-9}$$

$$\ell_2(t) = \frac{(t-1)(t+2)}{(4-1)(4+2)} = \frac{(t-1)(t+2)}{18}$$

$$\ell_3(t) = \frac{(t-1)(t-4)}{(-2-1)(-2-4)} = \frac{(t-1)(t-4)}{-18}$$

Thus

$$p(t) = 3\frac{(t-4)(t+2)}{-9} + 2\frac{(t-1)(t+2)}{18} + 8\frac{(t-1)(t-4)}{-18}$$

$$p(t) = \frac{(t-4)(t+2)}{-3} + \frac{(t-1)(t-2)}{9} + 4\frac{(t-1)(t-4)}{-9}$$

Please do not expand out or collect these terms.

Given an approximation ξ of the exact value x the absolute error is

$$E_{abs} = |\xi - x|$$

and the relative error is

$$E_{rel} = \frac{E_{abs}}{|x|}.$$

Suppose $\xi = 4.6$ and $x = 4.589$ then

$$E_{abs} = |4.6 - 4.589| = 0.011.$$

and

$$E_{rel} = \frac{0.011}{4.589} \approx 0.002397\ldots$$

The vector p-norm is defined as

$$\|x\|_p = \begin{cases} \sqrt[p]{\sum_{i=1}^{n} |x_i|^p} & \text{for } p \in [1, \infty) \\ \max\{|x_i| : i = 1, \dots, n\} & \text{for } p = \infty . \end{cases}$$

Given the vector $x = \begin{bmatrix} 1 \\ 2 \\ 3 \\ -1 \\ 1 \end{bmatrix}$

compute $\|x\|_p$ for $p = 1, 2$ and ∞.

$$\|x\|_1 = \sum_{i=1}^{5} |x_i| = 1 + 2 + 3 + 1 + 1 = 8$$

$$\|x\|_2 = \sqrt{\sum_{i=1}^{5} |x_i|^2} = \sqrt{1+4+9+1+1} = \sqrt{16} = 4 .$$

$$\|x\|_\infty = \mathrm{Max}(1, 2, 3, 1, 1) = 3$$