

The Euler-Masheroni Constant

Let H_n be the sum of the first n terms of the harmonic series

$$H_n = \sum_{k=1}^n \frac{1}{k}$$

```
In [2]: function H(n)
        r=0
        for k=1:n
            r=r+1/k
        end
        return r
    end
```

Out[2]: H (generic function with 1 method)

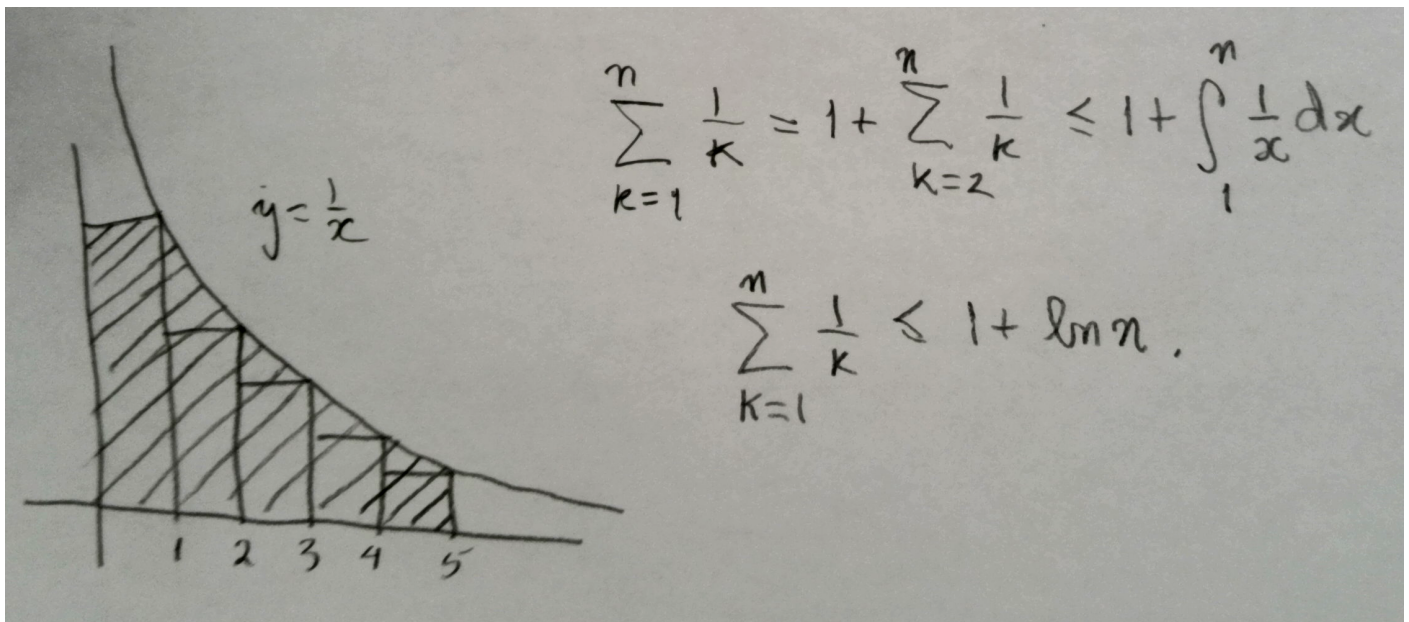
```
In [3]: H(10)
```

Out[3]: 2.9289682539682538

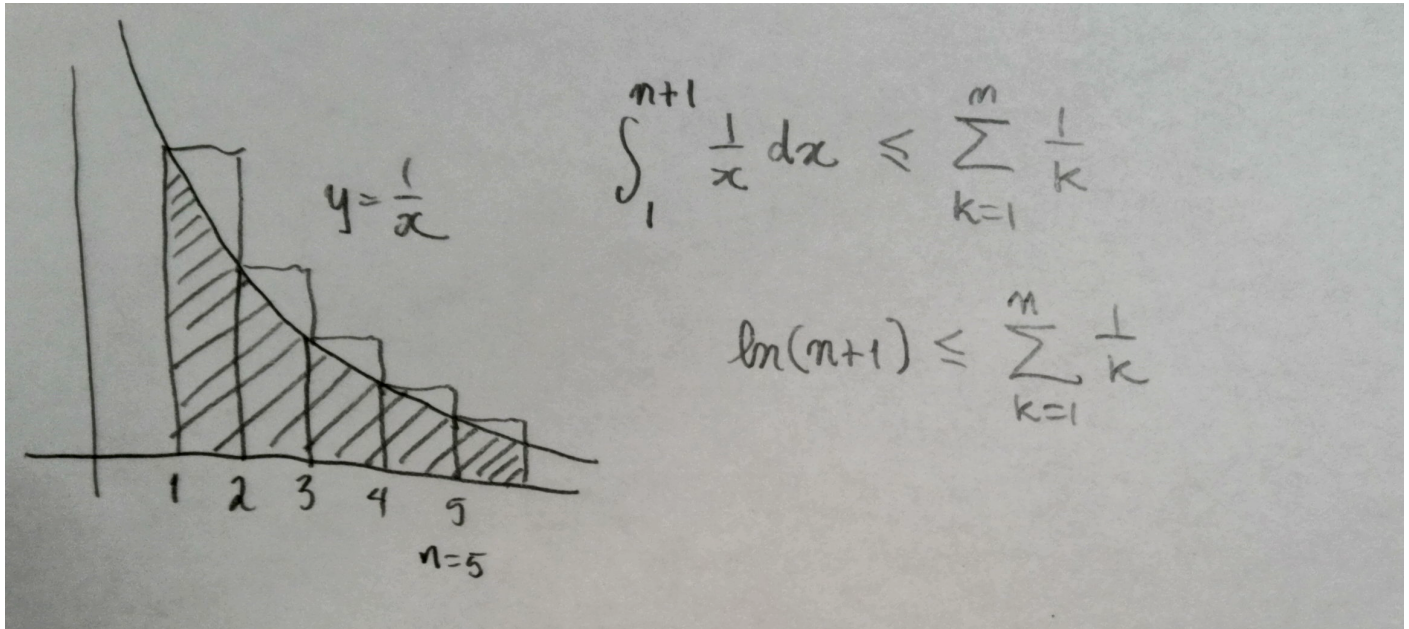
A simple Calculus argument shows that

$$\ln(1+n) \leq H_n \leq 1 + \ln n$$

The upper bound may be obtained as



while the lower as



The Euler-Mascheroni constant γ is given by the limit

$$\gamma = \lim_{n \rightarrow \infty} (H_n - \ln n) \approx 0.5772$$

```
In [4]: for j=1:20
        n=2^j
        println("n=$n, gamma=", H(n) - log(n))
        end
```

```
n=2, gamma=0.8068528194400547
n=4, gamma=0.6970389722134425
n=8, gamma=0.6384156011773068
n=16, gamma=0.6081402709892125
n=32, gamma=0.5927592926367935
n=64, gamma=0.585007820346096
n=128, gamma=0.5811168286695567
n=256, gamma=0.5791675183377185
n=512, gamma=0.5781919095102124
n=1024, gamma=0.5777038666786787
n=2048, gamma=0.5774597856583172
n=4096, gamma=0.5773377302469473
n=8192, gamma=0.577276698815977
n=16384, gamma=0.5772461821692296
n=32768, gamma=0.5772309236129889
n=65536, gamma=0.57722329427666
n=131072, gamma=0.5772194795937917
n=262144, gamma=0.5772175722491291
n=524288, gamma=0.5772166185756209
n=1048576, gamma=0.5772161417378925
```

In []: