This is an example of finite differences

Consider an equally space grid $x_j = x_0 + hj$ where $h > 0$.

In [1]:
```
x0=-1
```
Out[1]:
```
-1
```

In [2]:
```
h=0.3
```
Out[2]:
```
0.3
```

In [3]:
```
J=12
```
Out[3]:
```
12
```

In [4]:
```
js=[1:J;]
```
Out[4]:
```
12-element Array{Int64,1}:
  1
  2
  3
  4
  5
  6
  7
  8
  9
 10
 11
 12
```

In [5]:
```
xs=x0.+h*js
```
Out[5]:
```
12-element Array{Float64,1}:
 -0.7
 -0.4
 -0.10000000000000009
  0.19999999999999996
  0.5
  0.7999999999999998
  1.1
  1.4
  1.6999999999999997
  2.0
  2.3
  2.5999999999999996
```

Define $f_j = f(x_j)$ where

$$f(x) = \frac{1}{1+x^2}.$$

In [6]:
```
f(x)=1/(1+x^2)
```

Out[6]:
```
f (generic function with 1 method)
```

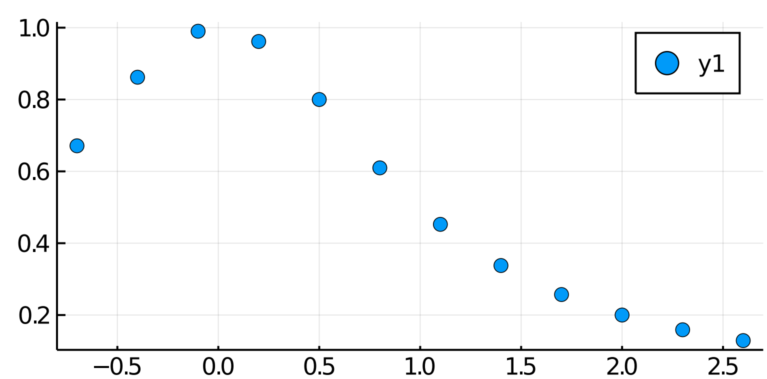
In [7]:
```
fs=f.(xs)
```

Out[7]:
```
12-element Array{Float64,1}:
 0.6711409395973155
 0.8620689655172413
 0.9900990099009901
 0.9615384615384615
 0.8
 0.6097560975609757
 0.45248868778280543
 0.33783783783783783
 0.25706940874035994
 0.2
 0.15898251192368842
 0.12886597938144334
```

In [8]:
```
using Plots
```

In [9]:
```
scatter(xs,fs,size=[400,200])
```

Out[9]:



Create a table of finite differences such that

$$T_{j,k+1} = \Delta^k f_j$$

for $k = 0, \ldots, n$ and $j = 1, \ldots, J - k$.

In [10]:
```
n=5
```

Out[10]:
```
5
```

In [11]:
```
T=zeros(J,n+1);
```

In [12]:
```
T[:,1]=fs;
```

In [13]:
```
T
```

Out[13]:
```
12×6 Array{Float64,2}:
 0.671141  0.0  0.0  0.0  0.0  0.0
 0.862069  0.0  0.0  0.0  0.0  0.0
 0.990099  0.0  0.0  0.0  0.0  0.0
 0.961538  0.0  0.0  0.0  0.0  0.0
 0.8       0.0  0.0  0.0  0.0  0.0
 0.609756  0.0  0.0  0.0  0.0  0.0
 0.452489  0.0  0.0  0.0  0.0  0.0
 0.337838  0.0  0.0  0.0  0.0  0.0
 0.257069  0.0  0.0  0.0  0.0  0.0
 0.2       0.0  0.0  0.0  0.0  0.0
 0.158983  0.0  0.0  0.0  0.0  0.0
 0.128866  0.0  0.0  0.0  0.0  0.0
```

In [14]:
```
for k=1:n
    for j=1:J-k
        T[j,k+1]=T[j+1,k]-T[j,k]
    end
end
```

Newton's forward difference formula for the interpolating polynomial is

$$f(x_j + \theta h) \approx \sum_{k=0}^{n} \binom{\theta}{k} \Delta^k f_j$$

In [15]:
```
function fp(j,theta)
    global n
    r=0.0
    b=1.0
    for k=0:n
#        println("k=$k")
        r=r+b*T[j,k+1]
        b=b*(theta-k)/(k+1)
    end
    return r
end
```

Out[15]:
```
fp (generic function with 1 method)
```

In [16]:
```
fp(2,-1)
```

Out[16]:
```
0.7577457729408739
```

In [17]:
```
fs[1]
```

Out[17]:
```
0.6711409395973155
```

In [18]:
```
p(x)=fp(1,(x-xs[1])/h)
```

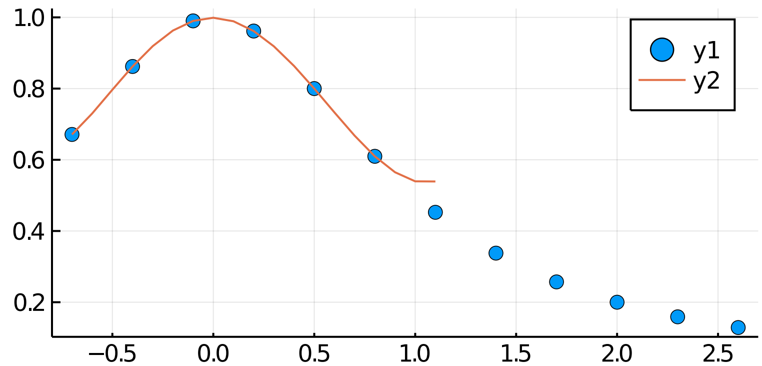
Out[18]:
```
p (generic function with 1 method)
```

In [19]:
```
xvals=[xs[1]:0.1:xs[7];];
```

In [20]:
```
yvals=p.(xvals);
```

In [21]:
```
plot!(xvals,yvals)
```

Out[21]:



In [ ]: