Idea, Use the spectral mapping theorem to find other eigenvalues other than just the one with largest magnitude...

$$\frac{y_{n+1} = Ax_n}{x_{n+1}} = \frac{y_{n+1}}{\|y_{n+1}\|}$$

Define a new matrix $B = A - \propto T$

How do the digenvalues of B compare with eigenvalues of A?

det & be an eigenvector of A with eigenvalue 2.

AZ= 25.

What about 13?

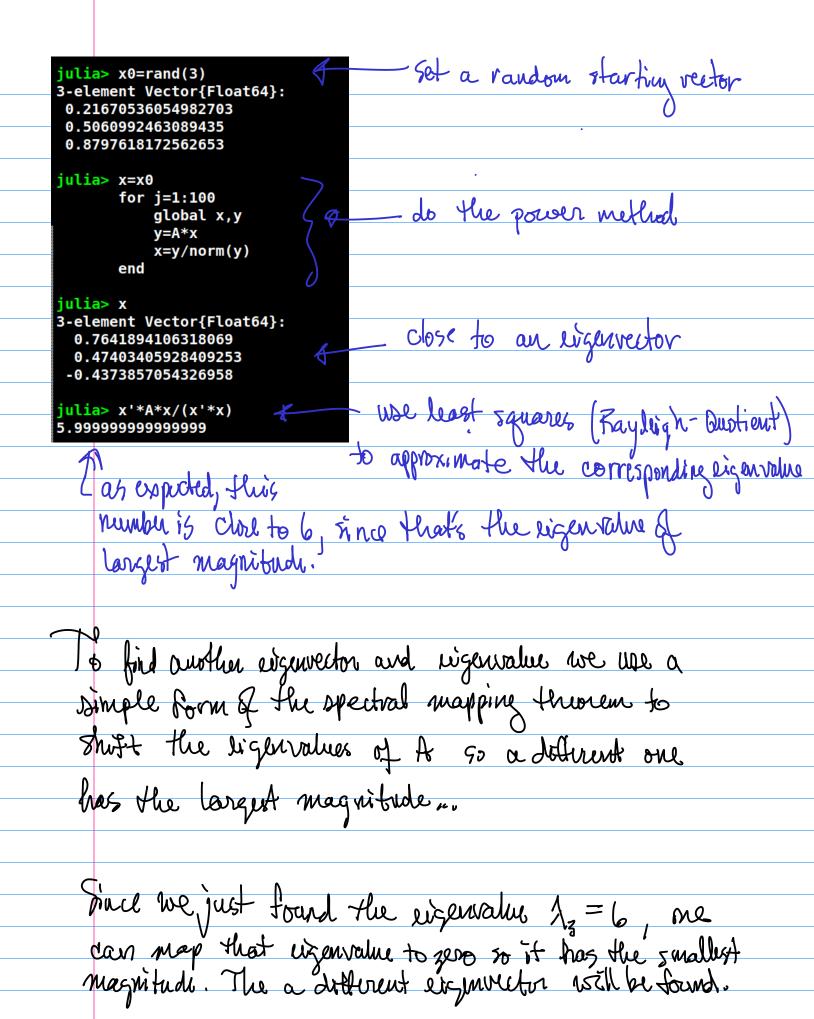
B== (A-aI)== A=- X=

Thruster & in also an eighnedor of B, but with shifted eightrales

Suppose the eigenvalues of A are given by 1=3, 1=4, 2=6 Digenrectors &, 32 33 Power iteration. your = Axn $Int_1 = \frac{y_{n+1}}{|y_{n+1}|}$ Then In "converges" to the eigenvector &s $\frac{x_n \cdot Ax_n}{x_n \cdot x_n} \Rightarrow \lambda_3 \quad a_3 \quad n \rightarrow \infty$

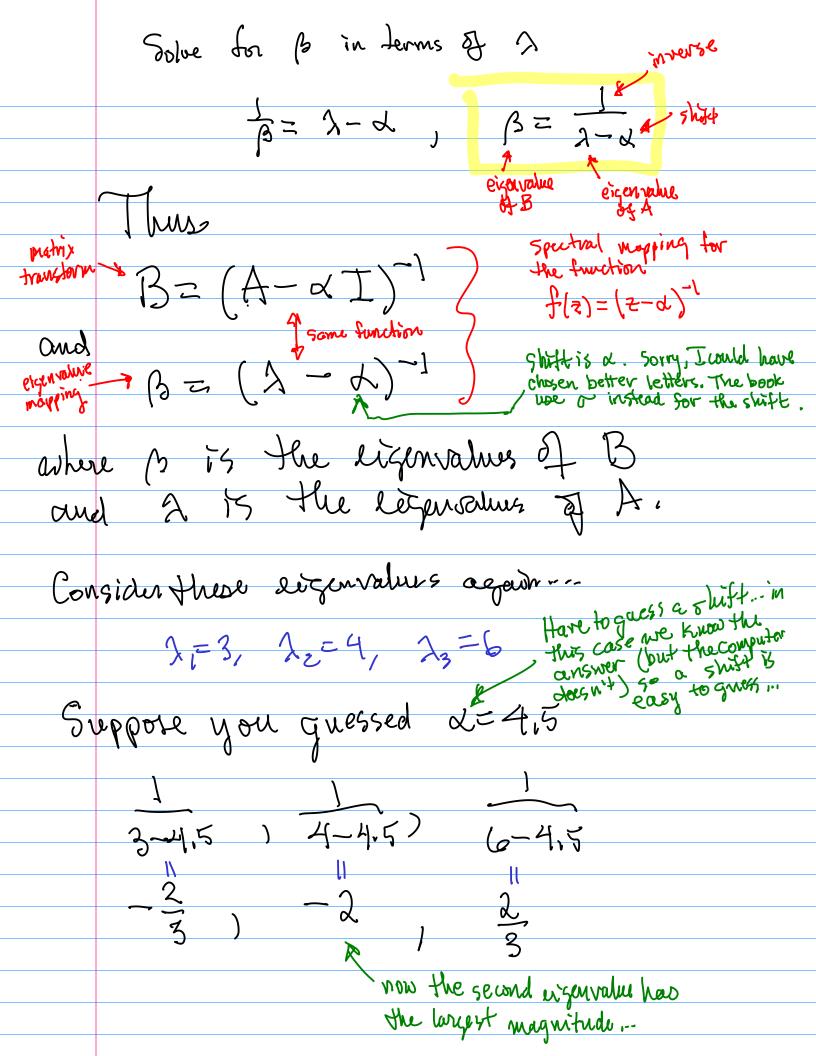
```
julia> using LinearAlgebra
                                         similarity transform
julia> S=rand(3,3) .- 0.5
3×3 Matrix{Float64}:
  0.0261003 0.480798
                        0.407647
 -0.0478098 -0.448344
                        0.252868
 0.137901 -0.390876
                       -0.233318
                                       Matrix with eigenvalues
julia> D=diagm([3,4,6])
3×3 Matrix{Int64}:
 0 4 0
                                         Matrix with same
eigenvalues
julia> A=S*D*inv(S)
3×3 Matrix{Float64}:
  5.21801 1.28406
                     0.025378
 1.01841 4.73558
                     0.408967
 -1.32488 -0.589416
                     3.04641
                                         use built-in function to
check eigenvalues...
julia> eigvals(A)
3-element Vector{Float64}:
3.000000000000001
```

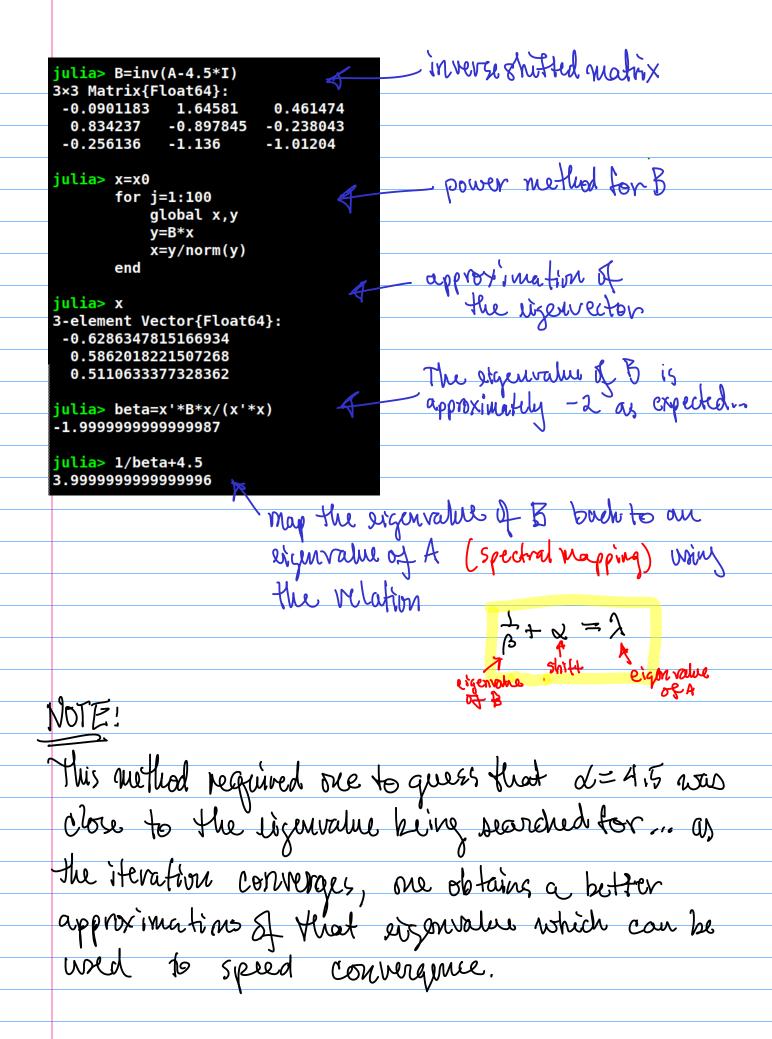
3.999999999999987

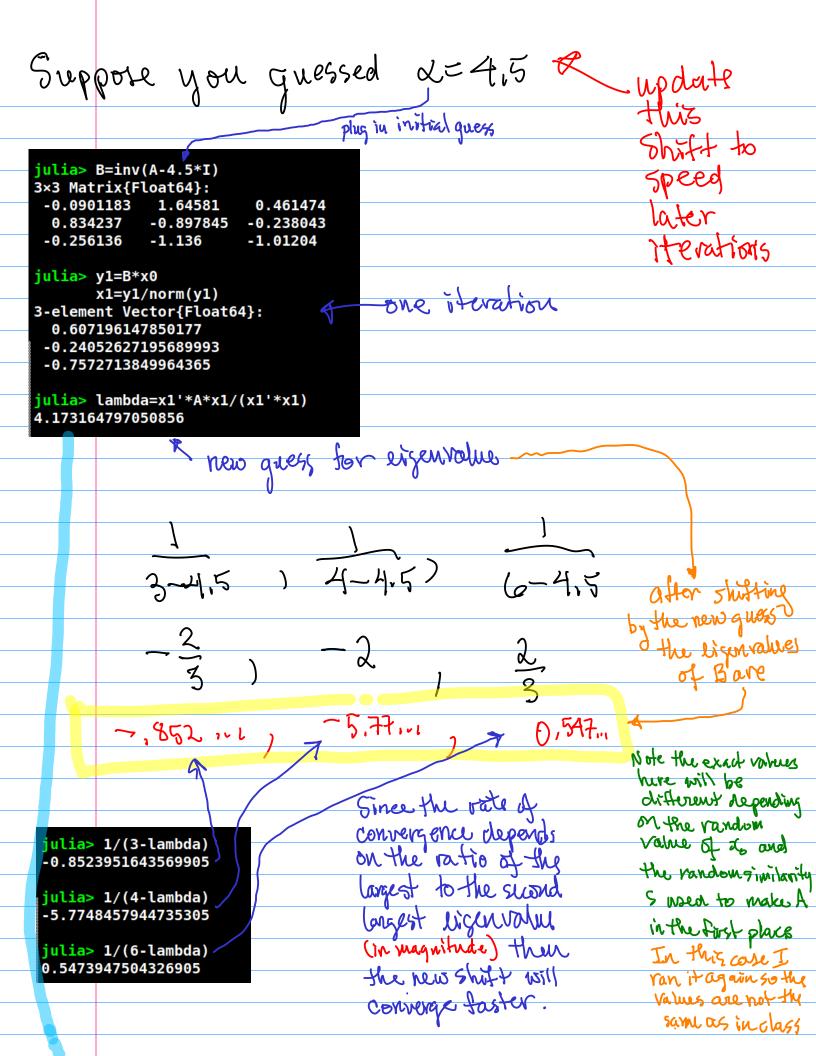


WShift by b ligenvalues of B one now the shifts B= A-6I ligen value R of lorgest magnitude Found this one which means -376=3x is the eigenvalue of the original matoix A. Shifted power method I The shifted matrix julia> B=A-6*I 3×3 Matrix{Float64}: -0.781987 1.28406 0.025378 1.01841 -1.26442 0.408967 -1.32488 -0.589416 -2.95359 this code block. Then press julia> x=x0 for j=1:100 global x,y y=B*x 🔨 to enter the editor and x=y/norm(y) end ahouge the Atoa B. julia> x Change this line 3-element Vector{Float64}: 0.1760333940369508 -0.32245321022423423 0.9300732075486947 The approximate vigor rector julia> beta=x'*B*x/(x'*x) -3.0 opproximation of the eigenvalue of B. Shaft it book to get the eigenvalue of A eigenvalue of B

	Shifted inverse power methodi.
	Define $B = (A - \propto I)^{-1}$
	How are the eigenvalues of B related to A?
That did work, so	Het & be an eigenvector & A with value 2. This Az=2z.
try aguin from the	
other was	$B\xi = (A-\lambda I)^{-1}\xi = \xi$
	Volue B. Thus B=33
	B= (A-dI)-1==B=
	Since B-1 = A-2I to identify an
	Thus $\xi = (A - \alpha I)\beta \xi$
	3 = BA3 - 2B3
	(1+dB) 3=BA3 That's the exercidus.
	$A = \left(\frac{1+d\beta}{\beta}\right)^{\frac{1}{2}}$
	Thus & is also an eigen vector of A with example 1+ 2B = 1 + 2 = 2
	B for shift eight value of A
	of B







un the new shift

```
<mark>julia> B=inv(A-4.173164797050856*I)</mark>
3×3 Matrix{Float64}:
 -1.05801
            3.85818
                      1.37653
  1.63198
           -3.08164
                     -1.08175
  0.390336 -2.92454
                     -1.9402
                              4 another iteration
julia> y2=B*x1
      x2=y2/norm(y2)
3-element Vector{Float64}:
 -0.5971833916006509
  0.5831282051682319
  0.550757199801891
julia> lambda=x2'*A*x2/(x2'*x2)
3.9629049493121054
               a better guess for the lightable.
 Confinue in this way results in accelerated
 Convergence similar (but port as fast) as Newton's nother.
                     use best shift so far
julia> B=inv(A-3.9629049493121054*I)
3×3 Matrix{Float64}:
  6.41948 -15.9679
                    -6.94759
 -5.38097
            15.3468
                      6.69918
            13.2132
 -5.81931
                      4.64386
                                        , another iteration
julia> y3=B*x2
       x3=y3/norm(y3)
3-element Vector{Float64}:
 -0.6289807133628148
  0.5875022772401237
  0.5091407825496536
julia> lambda=x3'*A*x3/(x3'*x3)
3.9999150022849284
              approximation of the eigenvalue is already quite good...
```

Our book calls this shifted-inverse iteration by a different name

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function Rayleigh-Quotient-Iteration (A, σ) $\vec{v} \leftarrow \text{Arbitrary}(n)$

for $k \leftarrow 1, 2, 3, ...$

late $\vec{v} \leftarrow (A - \sigma I_{n \times n})^{-1} \vec{v}$ $\vec{v} \leftarrow \vec{v}/\|\vec{w}\|$ $\sigma \leftarrow \frac{\vec{v}^\top A \vec{v}}{\|\vec{v}\|_2^2}$ This is the bast-square approximation of the excendence.